Chapter 3 roadmap:

- Propositions, booleans, logical equivalence. §3.(1 & 2) (last week Friday)
- Conditional propositions, conditional expressions. §3.3 (Monday)
- Arguments and predicates. §3.(5 & 6) (**Today**)
- Predicates and quantification. §3.(6 & 7) (Friday)
- (Begin proofs next week)

Today:

- Define arguments
- Consider known argument forms
- Practice verifying argument forms (Game 2)
- Begin predicates

Valid argument

If it is Monday, then it is raining
It is Monday.
Therefore it is raining.

$p \to q$
p
$\therefore q$

premise		premise	conclusion	
p	q	$p \to q$	q	
T	T	T	T	critical row
T	F	F	F	
F	T	T	T	
F	F	T	F	

Invalid argument

If it is raining, then there are clouds
There are clouds.
Therefore it is raining.

$p \rightarrow q$
q
$\therefore p$

	premise	*premise*	*conclusion*	
p	q	$p \rightarrow q$	p	
T	T	T	T	*critical row*
T	F	F	T	
F	T	T	F	*critical row*
F	F	T	F	

Alternate definition of validity

Valid argument

p	q	$p \to q$	$(p \wedge (p \to q)) \to q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Invalid argument

p	q	$p \to q$	$(q \wedge (p \to q)) \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Modus tollens

If it is spring, then the daffodils bloom.
The daffodils aren't blooming.
Therefore it is not spring.

p	q	$p \to q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Modus Ponens

$p \to q$

p

$\therefore q$

Modus Tollens

$p \to q$

$\sim q$

$\therefore \sim p$

Generalization

p

$\therefore p \vee q$

Specialization

$p \wedge q$

$\therefore p$

Elimination

$p \vee q$

$\sim p$

$\therefore q$

Transitivity

$p \to q$

$q \to r$

$\therefore p \to r$

Division into cases

$p \vee q$

$p \to r$

$q \to r$

$\therefore r$

Contradiction

$p \to F$

$\therefore \sim p$

Arguments in literature

Elmination:

> *If anyone knows anything about anything, it's Owl who knows something about something, or my name isn't Winnie-the-Pooh. Which it is. So there you have it.* A. A. Milne, *Winnie-the-Pooh*, Ch 4.

Division into cases:

> *Soon her eye fell on a little glass box that was lying under the table: she opened it, and found in it a very small cake, on which the words "EAT ME" were beautifully marked in currants. "Well, I'll eat it," said Alice, "and if it makes me grow larger, I can reach the key; and if it makes me grow smaller, I can creep under the door; so either way I'll get into the garden, and I don't care which happens!"* Lewis Carroll, *Alice's Adventures in Wonderland*, Ch 1.

Proof by contradiction

$p \to F$
$\therefore \sim p$

p	$p \to F$	$\sim p$	
T	F	F	
F	T	T	*critical row*

Restore us to yourself, O LORD, *that we may be restored. Renew our days as of old—unless you have utterly rejected us, and you remain exceedingly angry with us.* *Lam 5:21–22 (ESV)*

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.1

(a) $t \to u$

(b) $p \vee \sim q$

(c) $p \to (u \to r)$

(d) q

(e) $\therefore t \to r$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.2

(a) $p \to t$

(b) $\sim (q \to t) \to w$

(c) $p \vee q$

(d) $\sim w$

(e) $\therefore t$

Mod Pon	**Mod Tol**	**Generalization**	**Specialization**	**Elimination**	**Transitivity**	**Div into cases**	**Contradiction**
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.8

(a) w

(b) $q \to r$

(c) $t \to s$

(d) $u \to s$

(e) $(\sim t \wedge \sim u) \to \sim w$

(f) $(s \vee y) \to (p \to q)$

(g) $\sim (p \to r) \vee x$

(h) $\therefore x$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.9

(a) $p \to q$

(b) x

(c) $\sim (p \vee w) \to r$

(d) $q \to u$

(e) $x \to t$

(f) $w \to u$

(g) $r \vee s$

(h) $r \to F$

(i) $\therefore t \wedge s \wedge u$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.10

(a) $u \to \sim p$

(b) $(\sim p \vee q) \to (r \to s)$

(c) $u \wedge \sim w$

(d) $t \to s$

(e) $(\sim t \wedge \sim r) \to w$

(f) $\therefore s$

Propositions:

- $3 < 5$
- It's Thursday and it is snowing.
- If $3 < 5$ then $12 < 67$.

Propositional forms:

- $p \wedge q$
- $p \to q$

Four ways to interpret/define the idea of a *predicate*

- A predicate is a proposition with a parameter.

$$x < 5 \qquad\qquad x \text{ is orange}$$

- A predicate is a function whose value is true or false.

$$P(x) = x < 5 \qquad\qquad Q(x) = x \text{ is orange}$$

- A predicate is a part of a sentence that complements a noun phrase to make a proposition.
  A pumpkin is orange.
- A predicate is a truth set

	$P : \mathbb{N} \to \mathbb{B}, P(x) = x < 5$	$Q(x) = x$ is orange
Truth set:	$\{1, 2, 3, 4\}$	{ pumpkin, fall leaves, orange juice, ... }

For next time:

Do Exercises 3.5.(3, 5, 9-13)

Read Section 3.7.

Take quiz

(Note that exercises from Section 3.6 (Predicates) will be included in the next assignment.)