Chapter 3 roadmap:

Propositions, booleans, logical equivalence. §3.(1 & 2) (last week Friday)

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- Conditional propositions, conditional expressions. §3.3 (Monday)
- Arguments and predicates. §3.(5 & 6) (Today)
- Predicates and quantification. §3.(6 & 7) (Friday)
- (Begin proofs next week)

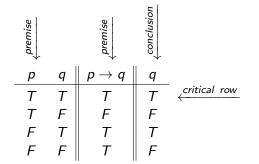
Today:

- Define arguments
- Consider known argument forms
- Practice verifying argument forms (Game 2)
- Begin predicates

# Valid argument

If it is Monday, then it is raining It is Monday. Therefore it is raining.

 $p \rightarrow q$ p $\therefore q$ 



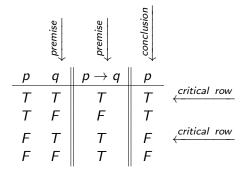
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# Invalid argument

If it is raining, then there are clouds There are clouds. Therefore it is raining.

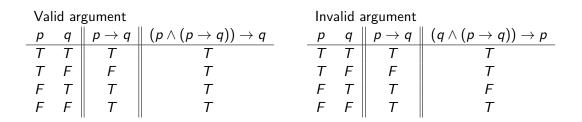
 $p \rightarrow q$ q $\therefore p$ 



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## Alternate definition of validity



# Modus tollens

If it is spring, then the daffodils bloom. The daffodils aren't blooming. Therefore it is not spring.

р	q		$\sim q$	$\sim p$
Т	Т		F	
Т	F	F	T	
F	Т	T	F	
F	F	T	T	Т

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$\begin{array}{l} \textbf{Modus Ponens} \\ p \rightarrow q \\ p \\ \therefore q \end{array}$	ModusTollens $p \rightarrow q$ $\sim q$ $\therefore \sim p$	Generalization p $\therefore p \lor q$	<b>Specialization</b> $p \land q$ $\therefore p$
Elimination $p \lor q$ $\sim p$ $\therefore q$	Transitivity $p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	Division into cases $p \lor q$ $p \to r$ $q \to r$ $\therefore r$	<b>Contradiction</b> $p \rightarrow F$ $\therefore \sim p$

## Arguments in literature

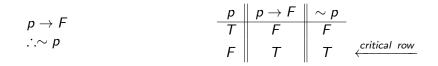
Elmination:

If anyone knows anything about anything, it's Owl who knows something about something, or my name isn't Winnie-the-Pooh. Which it is. So there you have it. A. A. Milne, Winnie-the-Pooh, Ch 4.

Division into cases:

Soon her eye fell on a little glass box that was lying under the table: she opened it, and found in it a very small cake, on which the words "EAT ME" were beautifully marked in currants. "Well, I'll eat it," said Alice, "and if it makes me grow larger, I can reach the key; and if it makes me grow smaller, I can creep under the door; so either way I'll get into the garden, and I don't care which happens!" Lewis Carroll, Alice's Adventures in Wonderland, Ch 1.

## Proof by contradiction



Restore us to yourself, O LORD, that we may be restored. Renew our days as of old—unless you have utterly rejected us, and you remain exceedingly angry with us. Lam 5:21-22 (ESV)

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$\begin{array}{c} Mod \ Pon \\ p \to q \end{array}$	$egin{array}{c} Mod \ Tol \ p  o q \end{array}$	Generalization	Specialization $p \land q$	Elimination $p \lor q$	Transitivity $p  ightarrow q$	Div into cases $p \lor q$	$\begin{array}{c} \textbf{Contradiction} \\ p \rightarrow F \end{array}$
р	$\sim q$	$\therefore p \lor q$	р	$\sim$ p	q  ightarrow r	ho  ightarrow r	.∴~ <i>p</i>
.'. q	.∴.~ <i>p</i>			∴. q	$\therefore p  ightarrow r$	q  ightarrow r	
3.9.1						∴ r	
(a) <i>t</i> –	→ u						
(b) <i>p</i> ∨	$\sim q$						
(c) p -	ightarrow ( $u  ightarrow$ r)						
(d) q							
(e) ∴ t	$t \rightarrow r$						

$\begin{array}{c} Mod \ Pon \\ p \to q \end{array}$	$egin{array}{c} Mod \ Tol \ p  o q \end{array}$	Generalization	Specialization $p \land q$	Elimination $p \lor q$	Transitivity $p  ightarrow q$	Div into cases $p \lor q$	$\begin{array}{c} \textbf{Contradiction} \\ p \rightarrow F \end{array}$
р	$\sim q$	$\therefore p \lor q$	р	$\sim$ p	q  ightarrow r	ho  ightarrow r	∴~ <i>p</i>
.'. q	.∴~ <i>p</i>			∴ q	$\therefore p  ightarrow r$	q  ightarrow r	
3.9.2						∴ r	
(a) p -	ightarrow t						
(b) $\sim$	(q  ightarrow t)  ightarrow	W					
(c) p \	/ q						
(d) $\sim$	W						
(e) ∴ :	t						

	$\begin{array}{c} Mod \ Tol \\ p \to q \end{array}$	Generalization <i>p</i>	Specialization $p \land q$	Elimination $p \lor q$	Transitivity $p  ightarrow q$	Div into cases $p \lor q$	$\begin{array}{c} \textbf{Contradiction} \\ p \rightarrow F \end{array}$
р	$\sim q$	$\therefore p \lor q$	∴. <i>p</i>	$\sim$ p	q  ightarrow r	p  ightarrow r	∴~ <i>p</i>
∴ q	∴~ <i>p</i>			.'. q	$\therefore p \rightarrow r$	q  ightarrow r	
3.9.8						:. r	
(a) <i>w</i>							
(b) q-	→ r						
(c) $t \rightarrow$	→ S						
(d) <i>u</i> –	≻ S						
(e) ( $\sim$	$t\wedge\sim u) ightarrow \gamma$	- W					
(f) (s \	(y)  ightarrow (p  ightarrow	· q)					
(g) $\sim$ (	$p \rightarrow r) \lor x$						
(h) ∴ <i>x</i>							

$egin{array}{l} Mod \ Pon \ p  ightarrow q \ p \end{array} \ p$	$egin{array}{l} Mod \ Tol \ p  o q \ \sim q \end{array}$	Generalization $p$ $\therefore p \lor q$	Specialization $p \land q$ $\therefore p$	Elimination $p \lor q$ $\sim p$	<b>Transitivity</b> $p \rightarrow q$ $q \rightarrow r$	Div into cases $p \lor q$ $p \to r$	<b>Contradiction</b> $p \rightarrow F$ $\therefore \sim p$
₽ ∴ q	∴~ <i>p</i>		<i>p</i>		$q \rightarrow r$ $\therefore p \rightarrow r$	q  ightarrow r	·· P
3.9.9						:. r	
(a) p-	$\rightarrow q$						
(b) x							
(c) $\sim$ (	$(p \lor w) \to r$						
(d) q-	→ u						
(e) <i>x</i> -	$\rightarrow t$						
(f) w -	$\rightarrow u$						
(g) r∨	5						
(h) r –	→ F						
(i) ∴ t	$\wedge s \wedge u$						

$ \begin{array}{l} \textbf{Mod Pon} \\ p \rightarrow q \\ p \\ \therefore q \end{array} $	$\begin{array}{c} \textbf{Mod Tol} \\ p \rightarrow q \\ \sim q \\ \therefore \sim p \end{array}$	$\begin{array}{c} \textbf{Generalization} \\ p \\ \therefore p \lor q \end{array}$	Specialization $p \land q$ $\therefore p$	Elimination $p \lor q$ $\sim p$ $\therefore q$	<b>Transitivity</b> $p \rightarrow q$ $q \rightarrow r$	Div into cases $p \lor q$ $p \to r$ $q \to r$	<b>Contradiction</b> $p \rightarrow F$ $\therefore \sim p$
3.9.10	P				$\therefore p \rightarrow r$	. r	
(a) <i>u</i> -	$ ightarrow \sim p$						
(b) ( $\sim$	$p \lor q)  ightarrow (r$	r  ightarrow s)					
(c) u∧	$\sim w$						
(d) t –	→ <i>S</i>						
(e) ( $\sim$	$t\wedge \sim r) \rightarrow$	W					
(f) ∴ s	;						

Propositions:

- ▶ 3 < 5
- It's Thursday and it is snowing.

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▶ If 3 < 5 then 12 < 67.

#### Propositional forms:

- ▶  $p \land q$
- ▶  $p \rightarrow q$

Four ways to interpret/define the idea of a predicate

A predicate is a proposition with a parameter.

x < 5 x is orange

A predicate is a function whose value is true or false.

$$P(x) = x < 5$$
  $Q(x) = x$  is orange

A predicate is a part of a sentence that complements a noun phrase to make a proposition.

A pumpkin is orange.

► A predicate is a truth set  $P: \mathbb{N} \to \mathbb{B}, P(x) = x < 5$ Truth set: {1,2,3,4} Q(x) = x is orange Q(x) = x is orange Q(x) = x is orange

#### For next time:

Do Exercises 3.5.(3, 5, 9-13)

Read Section 3.7.

Take quiz

(Note that exercises from Section 3.6 (Predicates) will be included in the next assignment.)

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