

## Chapter 3 outline:

- ▶ Propositions, booleans, logical equivalence. §3.(1 & 2) (last week Friday)
- ▶ Conditional propositions, conditional expressions. §3.3 (**Today**)
- ▶ Arguments and predicates. §3.(5 & 6) (Wednesday)
- ▶ Predicates and quantification. §3.(6 & 7) (Friday)
- ▶ (Begin proofs next week)

## Today:

- ▶ Highlight most important parts of conditionals
- ▶ Highlight most confusing parts of conditionals
- ▶ Observe programming connections
- ▶ Get a head start on arguments

$p$	$q$	$p \wedge q$	$p \vee q$	$\sim p$	$\sim p \vee q$	$p \rightarrow q$
$T$	$T$	$T$	$T$	$F$	$T$	$T$
$T$	$F$	$F$	$T$	$F$	$F$	$F$
$F$	$T$	$F$	$T$	$T$	$T$	$T$
$F$	$F$	$F$	$F$	$T$	$T$	$T$

$p$

$q$

If 12 divides 36 evenly, then 3 divides 72 evenly.

If  $3 < 72$ , then 3 divides 72 evenly.

If 12 divides 36 evenly, then  $72 < 3$ .

If  $72 < 3$ , then 3 divides 72 evenly.

If  $72 < 3$ , then 12 divides 3 evenly.

T	S	R	Q	P
K	L	M	N	O
J	I	H	G	F
E	D	C	B	A

1. Bob passed through *P*.
2. Bob passed through *N*.
3. Bob passed through *M*.
4. If Bob passed through *O*, then Bob passed through *F*.
5. If Bob passed through *K*, then Bob passed through *L*.
6. If Bob passed through *L*, then Bob passed through *K*.

“If Fred was at the dock at midnight, then he’s the murderer.”

“If it’s raining at home and the windows are still open, then water is coming in.”

“If I were John and John were me, then he’d be six and I’d be three.” — A. A. Milne

“If the dryer is finished, then unload it.”

“If you finish your spinach, then I will give you some cake.”

“If it rains tomorrow, the zucchini will sprout.”

An even degree is a **necessary condition** for a polynomial to have no real roots .  
*means*

If a polynomial function has no real roots, then it has an even degree.

A positive global minimum is a **sufficient condition** for a polynomial to have no real roots  
*means*

If a polynomial function has a positive global minimum, then it has no real roots.

Values all of the same sign is a **necessary** and **sufficient** condition for a polynomial to have no real roots.  
*means*

A polynomial function has values all of the same sign if and only if the function has no real roots.

		(original)					
$p$	$q$	conditional	converse	inverse	contrapositive	negation	biconditional
		$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	$p \wedge \sim q$	$p \leftrightarrow q$
$T$	$T$	$T$	$T$	$T$	$T$	$F$	$T$
$T$	$F$	$F$	$T$	$T$	$F$	$T$	$F$
$F$	$T$	$T$	$F$	$F$	$T$	$F$	$F$
$F$	$F$	$T$	$T$	$T$	$T$	$F$	$T$

With respect to the conditional proposition  
*If the jar is open, then the cookies are gone.*

identify each of the following propositions.

- ▶ The jar is open.
- ▶ The cookies are gone.
- ▶ If the cookies are gone, then the jar is open.
- ▶ If the jar is not open, then the cookies are not gone.
- ▶ If the cookies are not gone, then the jar is not open.



# Valid argument

If it is Monday, then it is raining  
It is Monday.  
Therefore it is raining.

$$p \rightarrow q$$

$$p$$

$$\therefore q$$

$p$	$q$	$p \rightarrow q$	$q$
$T$	$T$	$T$	$T$
$T$	$F$	$F$	$F$
$F$	$T$	$T$	$T$
$F$	$F$	$T$	$F$

Annotations:  
- Above the first column:  $p$  (premise)  
- Above the second column:  $q$  (premise)  
- Above the third column:  $p \rightarrow q$  (premise)  
- Above the fourth column:  $q$  (conclusion)  
- An arrow labeled "critical row" points to the second row (where  $p$  is true and  $q$  is false).

# Invalid argument

If it is raining, then there are clouds  
There are clouds.  
Therefore it is raining.

$p \rightarrow q$

$q$

$\therefore p$

$p$	$q$	$p \rightarrow q$	$p$	
$T$	$T$	$T$	$T$	$\leftarrow$ critical row
$T$	$F$	$F$	$T$	
$F$	$T$	$T$	$F$	$\leftarrow$ critical row
$F$	$F$	$T$	$F$	

# Modus tollens

If it is spring, then the daffodils bloom.  
The daffodils aren't blooming.  
Therefore it is not spring.

$p$	$q$	$p \rightarrow q$	$\sim q$	$\sim p$
$T$	$T$	$T$	$F$	
$T$	$F$	$F$	$T$	
$F$	$T$	$T$	$F$	
$F$	$F$	$T$	$T$	$T$

### Modus Ponens

$$p \rightarrow q$$
$$p$$
$$\therefore q$$

### Modus Tollens

$$p \rightarrow q$$
$$\sim q$$
$$\therefore \sim p$$

### Generalization

$$p$$
$$\therefore p \vee q$$

### Specialization

$$p \wedge q$$
$$\therefore p$$

### Elimination

$$p \vee q$$
$$\sim p$$
$$\therefore q$$

### Transitivity

$$p \rightarrow q$$
$$q \rightarrow r$$
$$\therefore p \rightarrow r$$

### Division into cases

$$p \vee q$$
$$p \rightarrow r$$
$$q \rightarrow r$$
$$\therefore r$$

### Contradiction

$$p \rightarrow F$$
$$\therefore \sim p$$

**For next time:**

*Do Exercises 3.4.(1-10), which is all of them.*

*Exercises 1-6 are on paper, Exercises 7-10 in a Jupyter notebook.*

*Read 3.(5 & 6)*

*Take quiz*