Chapter 4 roadmap:

- Subset proofs (day)
- Set equality and emptiness proofs (Wednesday)
- Conditional and biconditional proofs (**Today**)
- Proofs about powersets (next week Monday)
- Review for Test 2 (next week Wednesday)
- Test 2, on Chapters 3 & 4 (next week Friday, Oct 18)

Today:

- Proofs of conditional propositions
- Proofs about numbers
- Proofs of biconditional propositions

General forms:

1. Facts (p)
   Set forms
   1. Subset $X \subseteq Y$
   2. Set equality $X = Y$
   3. Set emptiness $X = \emptyset$
2. Conditionals ($p \to q$)
3. Biconditionals ($p \leftrightarrow q$)

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

To prove $p \to q$

Suppose p

$\ldots$

q

$p \to q$

An integer x is *even* if $\exists\ k \in \mathbb{Z} \mid x = 2k$.

An integer x is *odd* if $\exists\ k \in \mathbb{Z} \mid x = 2k + 1$.

"Axiom 3." If $x, y \in \mathbb{Z}$, then $x + y \in \mathbb{Z}$. (*Closure of addition*)

"Axiom 4." If $x, y \in \mathbb{Z}$, then $x \cdot y \in \mathbb{Z}$. (*Closure of multiplication*)

"Axiom 5." If $x \in \mathbb{Z}$, then x is even iff x is not odd.

$\forall\ x, y \in \mathbb{Z}$, $x \mid y$ (read, "x divides y") if $\exists\ k \in \mathbb{Z} \mid x \cdot k = y$.

Note that $y/x = k$ or $\frac{y}{x} = k$ or $\begin{array}{r} k \\ x \overline{)\,y} \end{array}$.

For next time:

Pg 162: 4.4.(1, 4, 5)
Pg 164: 4.5.(2 & 5)

Review 1.8, especially Ex 1.8.14
Skim 4.7

Take quiz