Chapter 4 roadmap:

- Subset proofs (last week Monday)
- Set equality and emptiness proofs (last week Wednesday)
- Conditional and biconditional proofs (Friday)
- ► Proofs about powersets (**Today**)
- Review for Test 2 (Wednesday)
- ► Test 2 (Friday)

Today: Case study of large proof (powersets)

- Review of powersets and their recursive structure
- Big result
- Warm-up proofs
- Proving the big result

Which are true?

$$\mathscr{P}(\emptyset) = \emptyset$$

$$\mathscr{P}(\emptyset) = \{\emptyset\}$$

$$\mathscr{P}(\emptyset) = \{\{\emptyset\}\}$$

$$\mathscr{P}(\emptyset) = \{\emptyset, \{\emptyset\}\}$$

$$\mathscr{P}(\{1\}) = \{1\}$$

$$\mathscr{P}(\{1\}) = \{\{1\}\}$$

$$\mathscr{P}(\{1\}) = \{\emptyset, \{1\}\}$$

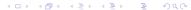
$$A\in\mathscr{P}(A)$$

$$A \subseteq \mathscr{P}(A)$$

$$A = \{a, b, c\} \qquad \mathscr{P}(A) = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}, \{b, c\}, \{b\}, \{c\}, \emptyset\}$$
$$A - \{a\} = \{b, c\} \qquad \mathscr{P}(A - \{a\}) = \{\{b, c\}, \{b\}, \{c\}, \emptyset\}$$

$$\{\{a\} \cup C \mid C \in \mathscr{P}(A - \{a\})\} = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\}\}$$

$$\mathcal{P}(A) = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}, = \{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} \\ \{b, c\}, \{b\}, \{c\}, \emptyset\} \cup \mathcal{P}(A - \{a\})$$



$$A = \{a, b, c\} \quad \mathscr{P}(A - \{a\}) = \{\{b, c\}, \{b\}, \{c\}, \emptyset\}$$

$$\{\{a\} \cup C \mid C \in \mathscr{P}(A - \{a\})\} = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\}\}$$

If $a \in A$, then $\mathscr{P}(A)$ consists in $\mathscr{P}(A - \{a\})$ and $\{C \cup \{a\} \mid C \in \mathscr{P}(A - \{a\})\}$

Corollary 4.12. If $a \in A$, then $\mathcal{P}(A - \{a\})$ and $\{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\}$ make a partition of $\mathcal{P}(A)$.

$$A \subseteq B$$
 iff $A \in \mathscr{P}(B)$

$$A \in \mathscr{P}(A)$$

$$\emptyset \in \mathscr{P}(A)$$

$$a \in A \text{ iff } \{a\} \in \mathscr{P}(A)$$

Warm-up proofs:

Theorem 4.7. If $\mathscr{P}(A) \subseteq \mathscr{P}(B)$, then $A \subseteq B$.

Exercise 4.7.1. If $B \subseteq A$, then $\mathscr{P}(B) - \mathscr{P}(A) = \emptyset$.

Roadmap

Corollary 4.12

 $\mathscr{P}(A - \{a\})$ and $\{C \cup \{a\} \mid C \in \mathscr{P}(A - \{a\})\}$ make a partition of $\mathcal{P}(A)$.

Theorem 4.11 / Exercise 4.7.6

Theorem 4.10.

$$\mathscr{P}(A - \{a\}) \cap \{C \cup \{a\} \mid C \in \mathscr{P}(A - \{a\})\} = \emptyset$$

 $\mathscr{P}(A-\{a\})\cup\{C\cup\{a\}\mid C\in\mathscr{P}(A-\{a\})\}=\mathscr{P}(A)$

Lemma 4.9.

Lemma 4.8.

$$\mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\}$$

$$\mathcal{P}(A) \subseteq$$

$$\mathcal{P}(A)$$

$$\mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\}$$

For next time:

Pg 174: 4.7.(1, 3, 4, 6)

Take quiz