Chapter 3:

- Propositions, booleans, logical equivalence. §3.(1 & 2) (**Today**)
- Conditional propositions, conditional expressions. §3.3 (next week Monday)
- Arguments and predicates. §3.(5 & 6) (next week Wednesday)
- Predicates and quantification. §3.(6 & 7) (next week Friday)
- (Begin proofs week-after)

Today:

- Retrospective on test and some recent homework
- Highlight main points of §3.1: Propositions, forms, etc
- Work through §3.2: Logical equivalences (Game 1)

A **proposition** is a sentence that is true or false, but not both.

*It is snowing and it is not Thursday.*

A **propositional form** is like a proposition but with content replaced by variables.

*p and not q*

$p \wedge \sim q$

$$\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$$

$$+ - \times \div$$

| $\times$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 |
| 2 | 0 | 2 | 4 | 6 |
| 3 | 0 | 3 | 6 | 9 |

$$\mathbb{B} = \{T, F\}$$

$$\vee \wedge \sim$$

| $\wedge$ | $T$ | $F$ |
|---|---|---|
| $T$ | $T$ | $F$ |
| $F$ | $F$ | $F$ |

| $\wedge$ | $T$ | $F$ |
|---|---|---|
| $T$ | $T$ | $F$ |
| $F$ | $F$ | $F$ |

| $\vee$ | $T$ | $F$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $F$ | $T$ | $F$ |

| $p$ | $\sim p$ |
|---|---|
| $T$ | $F$ |
| $F$ | $T$ |

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $F$ |

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $T$ |
| $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ |

| $p$ | $q$ | $p \wedge q$ | $p \vee q$ | $\sim p$ |
|---|---|---|---|---|
| $T$ | $T$ | $T$ | $T$ | $F$ |
| $T$ | $F$ | $F$ | $T$ | $F$ |
| $F$ | $T$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ | $F$ | $T$ |

Evaluate (to $T$ or $F$) this logical expression:

$(T \wedge (\sim F \vee F)) \wedge (T \wedge T)$

Evaluate (to $T$ or $F$) this logical expression:

$(T \vee F) \wedge \sim (F \wedge T)$

Evaluate (to $T$ or $F$) this logical expression:

$(F \vee F \vee T) \wedge (\sim T \wedge F)$

| $p$ | $q$ | $\sim p$ | $\sim q$ | $p \wedge q$ | $\sim (p \wedge q)$ | $\sim p \vee \sim q$ |
|---|---|---|---|---|---|---|
| $T$ | $T$ | $F$ | $F$ | $T$ | $F$ | $F$ |
| $T$ | $F$ | $F$ | $T$ | $F$ | $T$ | $T$ |
| $F$ | $T$ | $T$ | $F$ | $F$ | $T$ | $T$ |
| $F$ | $F$ | $T$ | $T$ | $F$ | $T$ | $T$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| Commutative laws: | $p \wedge q$ | $\equiv$ | $q \wedge p$ | $p \vee q$ | $\equiv$ | $q \vee p$ |
| Associative laws: | $(p \wedge q) \wedge r$ | $\equiv$ | $p \wedge (q \wedge r)$ | $(p \vee q) \vee r$ | $\equiv$ | $p \vee (q \vee r)$ |
| Distributive laws: | $p \wedge (q \vee r)$ | $\equiv$ | $(p \wedge q) \vee (p \wedge r)$ | $p \vee (q \wedge r)$ | $\equiv$ | $(p \vee q) \wedge (p \vee r)$ |
| Absorption laws: | $p \wedge (p \vee q)$ | $\equiv$ | $p$ | $p \vee (p \wedge q)$ | $\equiv$ | $p$ |
| Idempotent laws: | $p \wedge p$ | $\equiv$ | $p$ | $p \vee p$ | $\equiv$ | $p$ |
| Double negative law: | $\sim\sim p$ | $\equiv$ | $p$ | | | |
| DeMorgan's laws: | $\sim (p \wedge q)$ | $\equiv$ | $\sim p \vee \sim q$ | $\sim (p \vee q)$ | $\equiv$ | $\sim p \wedge \sim q$ |
| Negation laws: | $p \vee \sim p$ | $\equiv$ | $T$ | $p \wedge \sim p$ | $\equiv$ | $F$ |
| Universal bound laws: | $p \vee T$ | $\equiv$ | $T$ | $p \wedge F$ | $\equiv$ | $F$ |
| Identity laws: | $p \wedge T$ | $\equiv$ | $p$ | $p \vee F$ | $\equiv$ | $p$ |
| Tautology and contradiction laws: | $\sim T$ | $\equiv$ | $F$ | $\sim F$ | $\equiv$ | $T$ |

Remember from high school algebra that there are "simplify" problems and "solve" problems.

- Simplify $3x(2+3x)^2+1$.

$$\begin{aligned} & 3x(2+3x)^2+1 \\ = \quad & 3x(4+12x+9x^2)+1 \\ = \quad & 12x+36x^2+27x^3+1 \\ = \quad & 27x^3+36x^2+12x+1 \end{aligned}$$

- Solve $12x = 57 - 7x$ for $x$.

$$\begin{aligned} 12x &= 57-7x \\ 19x &= 57 \\ x &= 3 \end{aligned}$$

Suppose we were to show that $\sim (\sim p \wedge q) \vee (p \vee \sim p) \equiv p \vee \sim q$.

**Do this:**

$$\begin{array}{lll} & \sim (\sim p \wedge q) \vee (p \wedge \sim p) & \\ \equiv & \sim (\sim p \wedge q) \vee F & \text{by negation law} \\ \equiv & \sim (\sim p \wedge q) & \text{by identity law} \\ \equiv & p \vee \sim q & \text{by De Morgan's} \end{array}$$

**Don't do this:**

$$\begin{array}{rcll} \sim (\sim p \wedge q) \vee (p \wedge \sim p) & \equiv & p \vee \sim q & \\ \sim (\sim p \wedge q) \vee F & \equiv & p \vee \sim q & \text{by negation law} \\ \sim (\sim p \wedge q) & \equiv & p \vee \sim q & \text{by identity law} \\ p \vee \sim q & \equiv & p \vee \sim q & \text{by De Morgan's} \end{array}$$

**For next time:**

*Do Exercises 3.2.(2, 4, 8-12)*

*You may redo Exercise 2.3.10 (*`interweave_list`*) and/or 2.3.17 (*`quick_sort`*)*

*Read 3.4*
*Take quiz*