Chapter 3 roadmap:

- Propositions, booleans, logical equivalence. §3.(1 & 2) (last week Monday)
- Conditional propositions and arguments. §3.(3 & 4) (last week Wednesday)
- Predicates and quantification. §3.(6 & 7) (last week Friday)
- Quantified arguments §3.8 (**today**)
- (Begin proofs on Friday)

Today:

- Think through nested (multiple) quantification (leftover from §3.7)
- Think through quantified argument forms
- Practice verifying validity of quantified argument forms (Game 3)

Common forms for propositions

$\forall\ x \in A, P(x)$

$\forall\ x \in A, P(x) \to Q(x)$

$\exists\ x \in A \mid P(x)$

Universal instantiation

$\forall\ x \in A,\ P(x)$

$a \in A$

$\therefore P(a)$

Universal generalization

Suppose $a \in A$

$\quad P(a)$

$\therefore \forall\ x \in A,\ P(x)$

Hypothetical conditional

Suppose p

$\quad q$

$\therefore p \to q$

Existential instantiation

$\exists\ x \in A \mid P(x)$

Let $a \in A \mid P(a)$

$\therefore a \in A \wedge P(a)$

Existential Generalization

$a \in A$

$P(a)$

$\therefore \exists\ x \in A \mid P(x)$

Hypothetical division into cases

$p \vee q$

Suppose p

$\quad r$

Suppose q

$\quad r$

$\therefore r$

Universal modus ponens

$\forall\ x \in A,\ P(x) \to Q(x)$

$a \in A$

$P(a)$

$\therefore Q(a)$

Universal modus tollens

$\forall\ x \in A,\ P(x) \to Q(x)$

$a \in A$

$\sim Q(a)$

$\therefore \sim P(a)$

3.8.4

a. $\forall\ x \in A,\ \ P(x) \wedge \sim Q(x)$
b. $\forall\ x \in A,\ \ x \in B$
c. $\forall\ x \in B,\ \ \sim Q(x) \rightarrow R(x)$
d. $\therefore \forall\ x \in A,\ \ R(x)$

3.8.5

a. $\forall\ x \in A,\ x \in B$
b. $\forall\ x \in B,\ \sim P(x)$
c. $\forall\ x \in A,\ Q(x) \to P(x)$
d. $\therefore \forall\ x \in A,\ \sim Q(x)$

(Extra # 1)

(a) $\forall\ y \in B, \exists\ x \in A \mid R(x, y)$

(b) $\forall\ x \in A, \forall\ y \in B, (P(x) \land R(x, y) \to Q(y))$

(c) $\therefore (\forall\ x \in A, P(x)) \to (\forall y \in B, Q(y))$

(Extra # 2)

(a) $\forall\ x \in A, P(x)$

(b) $\forall\ x \in A, x \in B \vee R(x)$

(c) $\forall\ y \in B, Q(y) \vee \sim P(y)$

(d) $\forall\ x \in A, R(x) \rightarrow Q(x)$

(e) $\therefore \forall x \in A, Q(x)$

(Extra # 3)

(a) $\forall\ x \in A, P(x) \rightarrow R(x)$

(b) $\exists\ x \in A \mid P(x)$

(c) $\forall\ x \in A, Q(x) \vee x \in B$

(d) $\forall\ x \in A, P(x) \rightarrow \sim Q(x)$

(e) $\therefore \exists\ y \in B \mid R(y)$

3.8.10

a. $\forall\ x \in A,\ \exists\ y \in B \mid P(x, y)$
b. $\forall\ y \in B,\ \ Q(y) \vee R(y)$
c. $\forall\ x \in A,\ \ y \in B,\ \ P(x, y) \rightarrow \sim Q(y)$
d. $\exists\ x \in A \mid S(x)$
e. $\therefore \exists\ y \in B \mid R(y)$

3.8.11

a. $\forall\, x \in A,\ x \in B \land x \in C$
b. $\forall\, x \in C,\ x \in D \lor x \in E$
c. $\forall\, x \in B,\ x \in D \to P(x)$
d. $\forall\, x \in B,\ x \in E \to Q(x)$
e. $\forall\, x \in B,\ P(x) \lor Q(x) \to R(x)$
f. $\therefore \forall\, x \in A, R(x)$

For next time:

Do Exercises 3.8.(6-9)
See Canvas for fully-parenthesized versions of Game 3 problems.

Read Section 4.1

Take quiz