Chapter 6 outline:

- Introduction, function equality, and anonymous functions (week-before Wednesday)
- Image and inverse images (week-before Friday)
- Function properties and composition (last week Monday)
- Map, reduce, filter (last week Wednesday)
- Cardinality (last week Friday)
- Countability (Monday)
- Review (**today**)
- Test 3, on Ch 5 & 6 (Friday)
- (Begin self-reference chapter next week Monday)

Today:

- What to expect for "relations" questions
- What to expect for programming questions
- What to expect for "functions" questions
- How can I help you?

Goals of this course

- Write programs in the functional style
- Think recursively
- Understand sets, relations, and functions so that they can model real-world (and abstract) information
- Use formal logic to prove mathematical propositions.

Concepts of Chapters 5 & 6

- What functions, relations, their properties, and their related terms mean
- How to apply formal definitions in proofs
- Modeling relations and functions in programs and applying those models to solve problems

Concepts

5.(1–3). The definitions of *relation*, *image*, *inverse*, *identity relation*, and *composition*. *reflexive*, *symmetric*, and/or *transitive*.

Testable skills

Write proofs involving relations and related terms.

If R is a symmetric relation on X, then $R - i_X$ is symmetric.

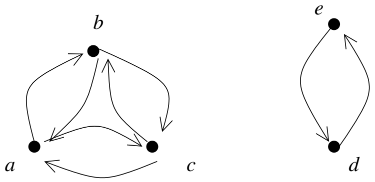
- A *relation* R from a set X to a set Y is a set of ordered pairs from X and Y; it is a subset of $X \times Y$.
- The *identity* relation on a set is $i_X = \{(x, x) \mid x \in X\}$.
- A relation R is *symmetric* if $\forall\ (x, y) \in R$, $(y, x) \in R$.

Concepts

5.(4 & 5). The definitions of *reflexive*, *symmetric*, *transitive*, *antisymmetric*, *transitive (and other) closure*, *partial order relation*, *total order relation*, and *topological sort*.

Testable skills

For a given concrete relation, determine which properties it has and find its transitive closure and a topological sort, if applicable.



a. Is this relation reflexive?
If **no**, then give a counterexample.

b. . . . symmetric?

c. . . . transitive?

d. . . . antisymmetric?

e. Draw the transitive closure of this relation.

f. Give a topological sort for the transitive closure of this relation or explain why one does not exist.

Concepts	Testable skills
5.(1 & 4). Relations represented as sets of pairs. Algorithms for computing image, inverse, composition, and closures.	Write functions that compute information about relations—that is, compute the image, inverse, composition, or closure of a relation, or test whether a relation is symmetric, transitive, or antisymmetric.
6.1. Finite functions represented as Python dictionaries; dictionary subscripting, dictionary inclusion (`in`), dictionary comprehensions, dictionary union (`&`).	Write functions that use or make Python dictionaries.
6.5. The `reduce` function as a means of applying an operation sequentially over items in a collection.	Write functions that call `reduce`.

Concepts

6.(1–4). The definitions of *function*, *function equality*, *image inverse image*, *one-to-one*, *onto*, and *composition*.

Testable skills

Write proofs involving functions and related terms.

If $f : X \to Y$, $A \subseteq Y$, and f is onto, then $A \subseteq f[f^{-1}[A]]$.

- f f is a function from a set X to a set Y and $A \subseteq X$, then the *image* of A under f is $f[A] = \{y \in Y \mid \exists\ a \in A \text{ such that } f(a) = y\}$
- ...and $B \subseteq Y$, then the *inverse image* of B under f is $f^{-1}[B] = \{x \in X \mid f(x) \in B\}$.

Concepts	**Testable skills**
6.(1–4). The definitions of *function*, *function equality*, *image inverse image*, *one-to-one*, *onto*, and *composition*.	Write proofs involving functions and related terms.

If $f : X \to Y$ and $g : Y \to Z$ are both one-to-one, then $g \circ f : X \to Z$ is one-to-one.

- If f is a function from a set X to a set Y, then f is *one-to-one* if $\forall\, x_1, x_2 \in X$, if $f(x_1) = f(x_2)$ then $x_1 = x_2$.
- f $f : X \to Y$ and $g : Y \to Z$, then the *composition* of f and g is the function $g \circ f = \{(x, z) \in X \times Z \mid z = g(f(x))\}$.

For next time:

Study for test...

There would be a section to read (first section of Chapter 7) but I doubt it will be ready in time...(But we will begin Chapter 7 on Mon, Nov 25.)