

P/S mop-up... and Graph unit

- ▶ Benford's law (Monday)
- ▶ Bayesian inference (Wednesday)
- ▶ A little more Bayesian inference; begin graph theory (**today**)
- ▶ Graph proofs (next week Monday)
- ▶ Graph isomorphisms (next week Wednesday)
- ▶ Varieties of graphs (next week Friday)
- ▶ Graphs as models of information (week-after Monday)

Today:

- ▶ Using Bayesian inference for parameter estimation
- ▶ Graph terminology
- ▶ Outline and goals of graph unit

Suppose we have a coin that has a head and tail but is weighted. We consider five ways in which the coin's behavior could be affected by this weighting: 100% probability of landing head, 75%, 50%, 25%, and 0%.

Initially we believe that each of these is equally probable. How is that belief updated after a sequence of flips?

Adapted from Dworsky, *Probably Not*, 2019 pg 205

Bayesian vs Frequentist Probability

The frequentist point of view is based on the following postulates:

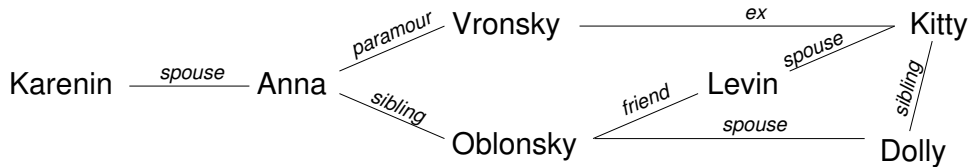
- F1 Probability refers to limiting relative frequencies. Probabilities are objective properties of the real world.*
- F2 Parameters are fixed, unknown constants.*
- F3 Statistical procedures should be designed to have well-defined long run frequency properties.*

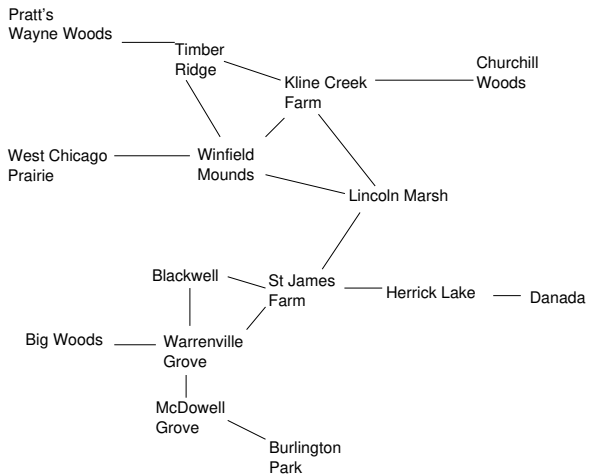
The Bayesian approach is based on the following postulates:

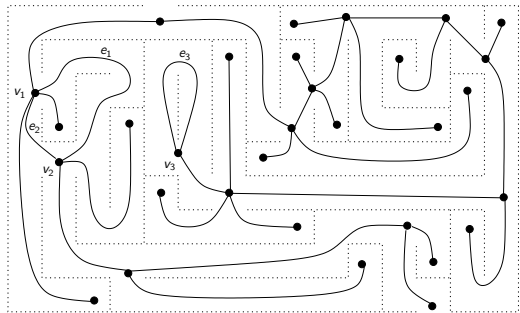
- B1 Probability describes degree of belief, not limiting frequency. “The probability that Albert Einstein drank a cup of tea on August 1, 1948 is .35” does not refer to any limiting frequency but reflects my strength of belief that the proposition is true.*
- B2 We can make probability statements about parameters, even though they are fixed constants.*
- B3 We make inferences about a parameter by producing a probability distribution for it.*

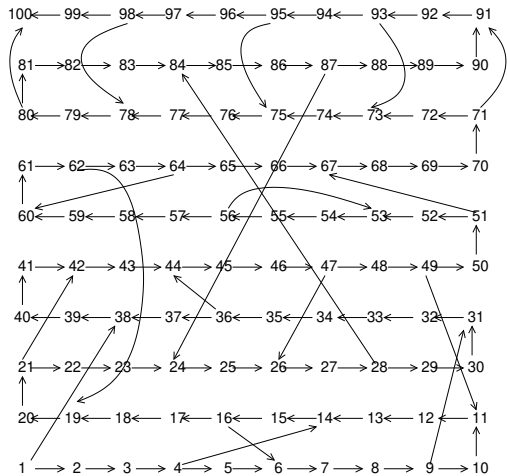
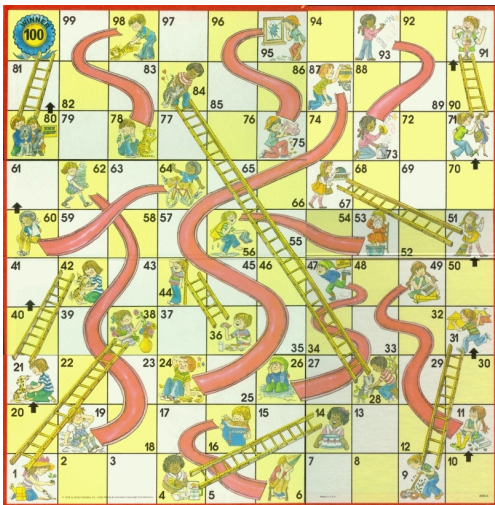
Wasserman, All of Statistics, 2004 pg 175-176, abridged.

$$\underbrace{P(A|B)}_{\text{posterior}} = \frac{\underbrace{P(B|A)}_{\text{likelihood}} \underbrace{P(A)}_{\text{prior}}}{\underbrace{P(B)}_{\text{marginal}}}$$









A *graph* $G = (V, E)$ is a pair of finite sets, a set V of *vertices* (singular *vertex*) and a set E of pairs of vertices called *edges*. We will typically write $V = \{v_1, v_2, \dots, v_n\}$ and $E = \{e_1, e_2, \dots, e_m\}$ where each $e_k = (v_i, v_j)$ for some v_i, v_j ; in that case, v_i and v_j are called *end points* of the edge e_k .

An edge (v_i, v_j) is *incident* on its end points v_i and v_j ; we also say that it *connects* them. If vertices v_i and v_j are connected by an edge, they are *adjacent* to one another.

If a vertex is adjacent to itself, that connecting edge is called a *self-loop*. If two edges connect the same two vertices, those edges are *parallel* to each other.

A graph is *undirected* if the edges indicate a symmetric relationship between the two endpoints: if there is an edge $e = (v_1, v_2)$, then v_1 is adjacent to v_2 and v_2 is adjacent to v_1 ; we could have described this edge as (v_2, v_1) . In a *directed graph*, sometimes abbreviated to *digraph*, the edges are not symmetric; rather, they have a direction and are displayed with arrowheads.

The *degree* $\deg(v)$ of a vertex v is the number of edges incident on the vertex, with self-loops counted twice. In a directed graph, we need to distinguish between the *in-degree* and *out-degree* of a vertex.

A *subgraph* of a graph $G = (V, E)$ is a graph $H = (W, F)$ where $W \subseteq V$ and $F \subseteq E$ (and, by definition of graph, for any edge $(v_i, v_j) \in F$, $v_i, v_j \in W$).

The *complement* of a simple graph $G = (V, E)$ is a graph $\bar{G} = (V, \bar{E})$ where for $v_i, v_j \in V$, $(v_i, v_j) \in \bar{E}$ if $(v_i, v_j) \notin E$; in other words, the complement has all the same vertices and all (and only) those possible edges that are not in G . A simple graph $G = (V, E)$ is *complete* if for all distinct $v_i, v_j \in V$, the edge $(v_i, v_j) \in E$.

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For next time:

Read Section 8.1 from DMFP; do Exercises 8.1.(1–10, 14)