Counting and combinatorics unit

- Cardinality and countability (**Today**)
- Combinations and permutations (Friday)
- The pigeonhole principle (next week Monday)

Today:

- Formal definition of cardinality
- Finite sets, infinite sets, and countability
- Countability proofs

Two sets X and Y have **the same cardinality as each other** if there exists a one-to-one correspondence from X to Y.

A set X is **finite** if there exists $n \in \mathbb{W}$ such that $\mathbb{Z}_n = \{0, 1, \ldots n-1\}$ has the same cardinality as X. We also say that the **cardinality** of X is n, that is, $|X| = n$.

A set X is **countably infinite** if it has the same cardinality as $\mathbb{W}$.

A set X is **countable** if it is either finite or countably infinite. Otherwise it is **uncountable**.

1. $\mathbb{W}$ has the same cardinality as $\mathbb{Z}$.
2. $(0, 1)$ (that is, $\{x \in \mathbb{R} \mid 0 < x < 1\}$) has the same cardinality as $\mathbb{R}$.
3. $(0, 1)$ is uncountable.
4. The set of functions from $\mathbb{N}$ to $\mathbb{Z}_{10}$ is uncountable.
5. The set of strings made from the symbols $\{0, 1\}$ is countably infinite. (We write the set of strings over $\{0, 1\}$ as $\{0, 1\}*$.)

For next time:

To turn in, prove

1. *The set of natural numbers* $\mathbb{N}$ *(positive integers) has the same cardinality as the set of perfect squares, that is* $\{1, 4, 9, 16, 25, \ldots\}$.
2. *The set* $[0, 1] = \{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$ *has the same cardinality as the set* $[0, 10] = \{x \in \mathbb{R} \mid 0 \leq x \leq 10\}$.

Take quiz