Discrete probability unit

- Sample spaces, events, axioms or probability (last week Wednesday and Friday)
- The sum and product rules; event-composition method (**Today**)
- Conditional, joint, and marginal probabilities (Wednesday)
- Independence (Friday)
- Random variables (next week Monday)
- Review (next weed Wednesday)
- Test (next week Friday)

Today:

- The sample-point method
- Conditional probability and independence
- The sum and product rules
- The event-composition method

Almost all examples and summaries are adapted from Dennis Wackerly et al, *Mathematical Statistics with Applications*, seventh edition, Brooks/Cole, 2008.

Consider two vehicles arriving at an intersection, each of which could turn left, turn right, or go straight.

a. List the basic outcomes in the sample space.
b. Assume all basic outcomes have equal probability. Compute the probability that at least one car turns left.
c. Compute the probability that at most one car turns.

Wackerly, pg 39

Sample-point method for computing the probability of an event:

1. Define the experiment so as to identify basic outcomes.
2. List the sample space (set of basic outcomes), Ω.
3. Determine probabilities for the basic outcomes such that $\forall\ x \in \Omega, 0 \leq P(\{x\}) \leq 1$ and $\sum_{x \in \Omega} P(\{x\}) = 1$.
4. Define the event of interest A, identifying the appropriate basic outcomes.
5. Find $P(A)$ by summing the probabilities of the basic outcomes in A.

Wackerly et al, pg 36

You are playing a card game with a standard 52-card deck that has been shuffled well. Another player draws a card.

a. What is the probability that the card is the queen of diamonds?
b. When the player drew the card, you noticed (in a reflection on the table) that the card was red and a face card. Now what is the probability that the card is the queen of diamonds?
c. In addition to seeing that reflection, it happens that you hold the king of hearts and jack of diamonds in your hand. Now what is the probability that the card drawn is the queen of diamonds?

Let A and B be events over a sample space. The **conditional probability** of A given B is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

(This is undefined if $P(B) = 0$.)

Moreover, A and B are **independent** if the following (equivalent) are true:

$$\begin{aligned} P(A \mid B) &= P(A) \\ P(B \mid A) &= P(B) \\ P(A \cap B) &= P(A)P(B) \end{aligned}$$

A shopkeeper says she has two new baby beagles to show you, but she doesn't know whether they're both male, both female, or one of each. You tell her that you want only a male, and she telephones the fellow who's giving them a bath.

"Is at least one a male?" she asks him. She receives a reply. "Yes!" she informs you with a smile.

What is the probability that the other one is a male?

Marilyn vos Savant

Theorem 1 (The product rule)

For all events A and B over a sample space,

$$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$$

Moreover, if A and B are independent,

$$P(A \cap B) = P(A)P(B)$$

Theorem 2 (The sum rule)

For all events A and B over a sample space,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Moreover, if A and B are mutually exclusive, $P(A \cap B) = 0$ *and*

$$P(A \cup B) = P(A) + P(B)$$

Suppose A and B are events over a sample space and $P(A) = .8$ and $P(B) = .7$.

a. Is it possible that $P(A \cap B) = .1$?
b. What is the least possible value for $P(A \cap B)$?
c. Is it possible that $P(A \cap B) = .77$?
d. What is the greatest possible value for $P(A \cap B)$?

Wackerly et al, pg 59

Of the items produced by a factory, 40% come from line I and 60% from line II. Line I has a defect rate of 8%, whereas line II has a defect rate of 10%. If an item is chosen at random from the day's production, what is the probability that it will *not* be defective?

Wackerly et al, pg 68

Event-composition method for computing the probability of an event:

1. Define the experiment, identifying a few basic outcome to clarify the definition.
2. Write an equation expressing an event of interest as a composition of two or more events using unions, intersections and complements.
3. Apply the additive and multiplicative laws of probability to the compositions in the previous step to compute the probability of the event of interest.

Wackerly et al, pg 64

A monkey is to demonstrate that she recognizes colors by tossing one red, one black, and one white ball into boxes of the same respective colors, one ball to a box. If the monkey has not learned the colors and merely tosses one ball into each box at random, then find the probabilities of the following results:

a. There are no color matches
b. There is exactly one color match

Wackerly et al, pg 67

For next time:

Do the exercises on the back of the handout (to turn in at class time).