Graph unit

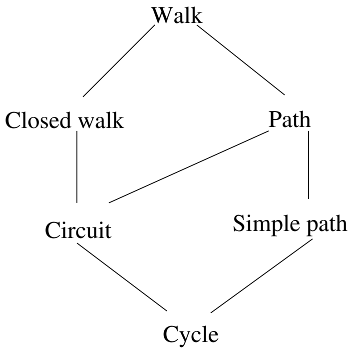
- A little more Bayesian inference; begin graph theory (last week Friday)
- Graph proofs (Monday)
- Graph isomorphisms (**today**)
- Varieties of graphs (Friday)
- Graphs as models of information (next week Monday)

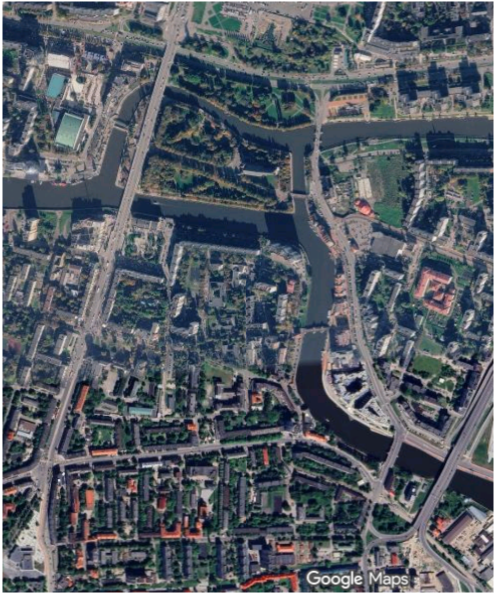
Today:

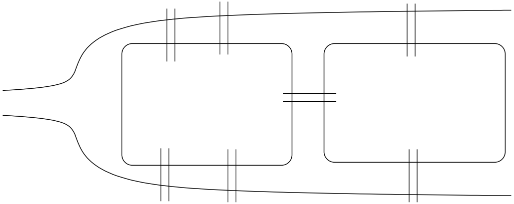
- "Euler circuit" proof
- Review Hamiltonian cycles
- Isomorphisms
- Isomorphism proofs

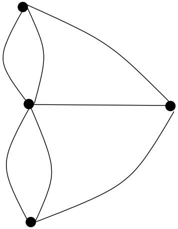
Adjectives	
Trivial	Having only one vertex and no edges.
Simple	Having no repeated *vertices* (except, possibly, the initial and terminal).
Closed	Having the same vertex as initial and terminal.

Nouns	
Walk	An alternating sequence of vertices and edges, each edge coming between its end points.
Path	A walk with no repeated *edge* (repeated vertices are ok).
Circuit	A closed path (no repeated edges, initial and terminal the same).
Cycle	A simple circuit (no repeated edges or vertices, except the initial and terminal, which are the same).




Google Maps



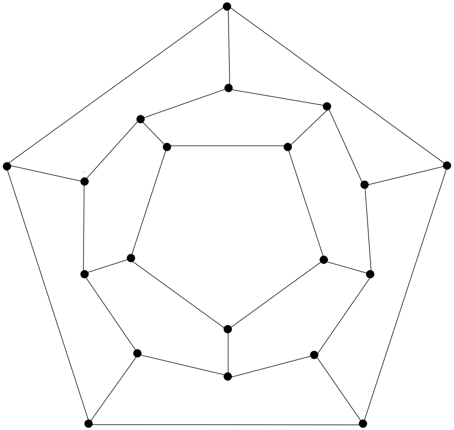


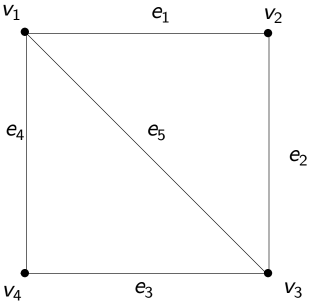
An *Euler circuit* of G is a circuit that contains every vertex and every edge. Since it is a circuit, this also means that an Euler circuit contains every edge exactly once. Vertices, however, may be repeated.

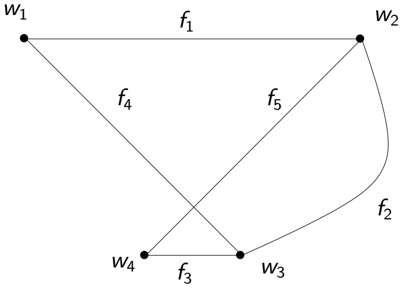
Theorem

If a graph $G = (V, E)$ has an Euler circuit, then every vertex of G has an even degree.

A *Hamiltonian cycle* is a cycle that includes every vertex in V. Since it is a cycle, this means that no vertex or edge is repeated; however, not all the edges need to be included.

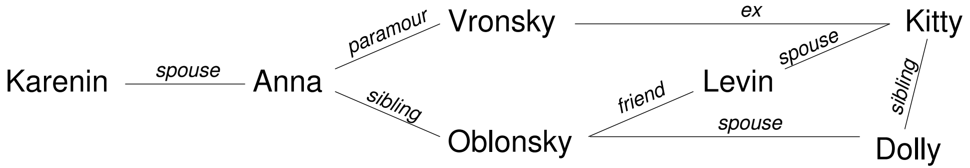


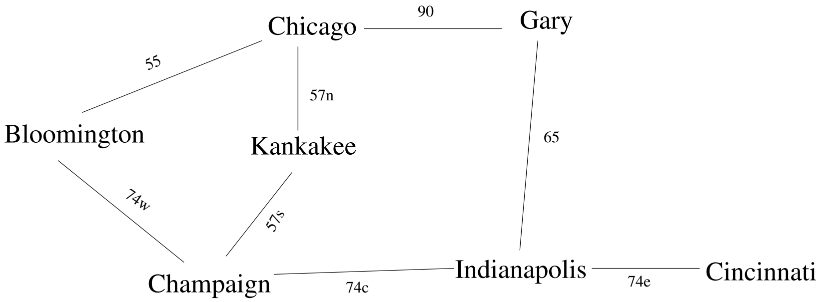




v	$\phi(v)$
v_1	w_2
v_2	w_4
v_3	w_3
v_4	w_1

e	$\psi(e)$
e_1	f_5
e_2	f_3
e_3	f_4
e_4	f_1
e_5	f_2


Karenin
spouse
Anna
paramour
Vronsky
ex
Kitty
sibling
Oblonsky
friend
Levin
spouse
sibling
spouse
Dolly


Chicago
90
Gary
55
57n
Bloomington
Kankakee
65
74w
57s
Champaign
74c
Indianapolis
74e
Cincinnati

Let $G = (V, E)$ and $H = (W, F)$ be graphs. G is **isomorphic** to H if there exist one-to-one correspondences $\phi : V \to W$ and $\psi : E \to F$ such that for all $v \in V$ and $e \in E$, v is an endpoint of e iff $\phi(v)$ is an endpoint of $\psi(e)$.

The two functions ϕ and ψ, taken together, are referred to as the **isomorphism** itself, that is, there exists an isomorphism, namely ϕ and ψ, between G and H.

An **isomorphic invariant** is a property that is preserved through an isomorphism. Formally, a predicate P is an isomorphic invariant if for any graphs G and H, if $P(G)$ and G is isomorphic to H, then $P(H)$.

Theorem

For any $k \in \mathbb{N}$, the proposition $P(G) =$ "G has a vertex of degree k" is an isomorphic invariant.

Theorem

Having a Hamiltonian cycle is an isomorphic invariant.

For next time:

Read Sections 8.4 from DMFP; do Exercises 8.4.(5 & 6)