Counting and combinatorics unit

- Cardinality and countability (last week Wednesday)
- Combinations and permutations (last week Friday)
- Various counting problems and techniques (**Today**)
- Begin probability unit (Wednesday)

Today:

- Tic-tac-toe states
- Balls and urns
- Permutations with indistinguishable items
- The inclusion/exclusion rule
- The pigeonhole principle

From the homework:

5. How many legal configurations are there of the game tic-tac-toe? Assume X goes first. For example,

,

X		X
	O	

, and

	X	O
X	O	
O	X	

are all legal, but

O	X	
O		

and

X	X	X
X	X	
	X	X

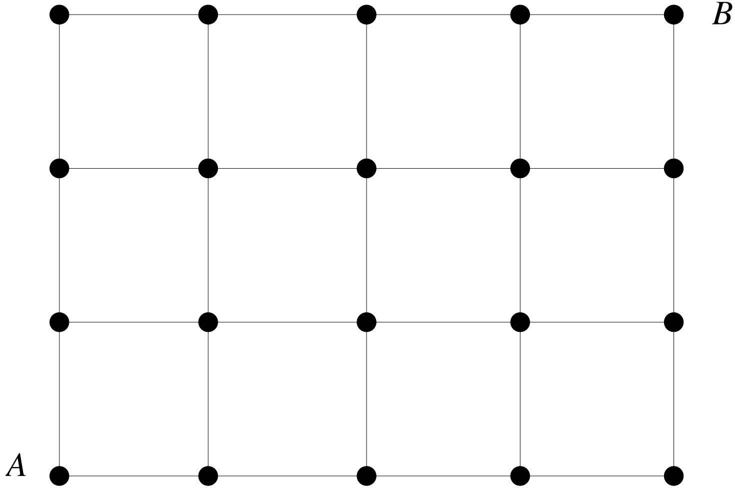
are not.

- Suppose there are n balls in a single urn, and assume that all the balls are easily distinguishable from each other. You grab balls from the urn one at a time, and at each draw you take note of the ball and *put it back in the urn*. These draws are ordered, but you could pull out the same ball multiple times. This is called **drawing with replacement**. How many different sequences of r draws are possible?
- Next imagine the same situation—n distinguishable balls in a single urns, drawn one a time, r times—but this time assume that we don't put the ball back in the urn after we draw it. Thus each ball may occur at most once in the sequence. How many distinct sequences are possible?
- Finally, imagine the same situation as before, but instead of pulling r balls one at a time, we grab all r balls in one draw. How many ways to draw the balls are possible?

A building is decorated by nine flags—four white, three red, and two blue. A covert organization wants to use these flags to signal secret messages, so that each distinguishable arrangement of flags indicates a different message. However, different flags of the same color are indistinguishable. How many different messages can be displayed using this system?

Adapted from Ross, *A First Course in Probability*, 1997 pg 4

How many paths are there from A to B using only steps up and right?



Adapted from Ross, *A First Course in Probability*, 1997 pg 18

A class of 47 students have experience in certain programming languages. 30 can program in Java, 18 can program in Python, and 26 can program in C. Moreover, 9 can program in both Java and Python, 16 in both Java and C, and 8 in both Python and C. (All know at least one of these languages.)

How many can program in all three?

Adapted from Epp, *Discrete Mathematics with Applications*, 2004 pg 329

- Humans have at most 150,000 hairs on their head. Wheaton has a population of 54,000. Chicago has a population of 2,700,000. Must there be two people in Wheaton with the same number of hairs on their head? Must there be two people in Chicago with the same number of hairs on their head?
- There are 9 students in this course. One of my other courses this semester has 19. In either course, are we guaranteed to know that at least two students in a class have the same birth month? What about birthday?
- Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. If 5 numbers are selected from A, must at least one pair of the integers in that subset sum to 9? If 4 numbers are selected, must at least one pair sum to 9? (Here "pair" means "combination/subset of size 2," not "ordered pair/2-tuple.")
- Show that all the decimal expansion of any rational number must repeat.

Adapted from Epp, *Discrete Mathematics with Applications*, 2004 pg 420–425

The pigeonhole principle. Let X and Y be finite sets and $f : X \to Y$. If f is one-to-one, then $|X| \leq |Y|$. Contrapositively, if $|X| > |Y|$, then f is not one-to-one.

The generalized form of the pigeonhole principle. Let X and Y are finite sets, $f : X \to Y$, and $k \in \mathbb{N}$. If $|X| > k|Y|$, then $\exists\ y \in Y$ such that $|f^{-1}[\{y\}]| \geq k + 1$ (that is, at least $k + 1$ elements are mapped to y).

Art credit: Sharon Dunbar '23

For next time:

Do the exercises on the back of the handout (to turn in at class time).