Discrete probability unit

- Sample spaces, events, axioms or probability (**Today**)
- Sum and product rules (Friday)
- The event-composition method (next week Monday)
- Conditional, joint, and marginal probabilities (next week Wednesday)
- Independence (next week Friday)
- . . .

Today:

- The basic definitions and axioms
- Theorems
- Examples

- Discrete Probability
  A. Sample spaces and events, and the axioms of probability
  B. Sum and product rules
  C. Conditional, joint, and marginal probability
  D. Discrete random variables
  E. Expected value and variance
  F. Common distributions
- Continuous Probability
  A. Continuous random variables
  B. Correlation
  C. Distributions
- Hypothesis testing
  A. Elements of statistical testing
  B. Z-tests and p-values
  C. Confidence intervals
- Interesting statistical results
  A. The central limit theorem
  B. The weak law of large numbers
  C. Benford's law
- Bayesian Reasoning
  A. Bayes's theorem
  B. Bayesian inference

Probability is a mathematical language for quantifying uncertainty.

Wasserman pg 3

One way of defining the probability of an event is in terms of its relative frequency. We suppose that an experiment is repeatedly performed under exactly the same conditions. For each event E, we define $n(E)$ to be the number of times in the first n repetitions of the experiment that the event E occurs. Then $P(E)$, the probability of the event E, is defined by

$$P(E) = \lim_{n\to\infty} \frac{n(E)}{n}$$

That is $P(E)$ is defined as the limiting proportion of time that E occurs.

Ross pg 30

However, there are also other uses of the term probability. For instance, we have all heard such statements as, "it is 90 percent probable that Shakespeare actually wrote Hamlet,"' or "the probability that Oswald acted alone in assassinating Kennedy is .8." The most simple and natural interpretation is that the probabilities referred to are measures of the individual's belief in the statements that he or she is making. *Ross pg 52*

The set of basic outcomes in the experiment is the *sample space*. An *event* is a set of basic outcomes from the sample space.
Let Ω be a sample space and $\mathcal{F} = \mathscr{P}(\Omega)$ be an event space; A *probability function* $P : \mathcal{F} \to [0, 1]$ fulfills the axioms of probability:

1. For all $A \in \mathcal{F}$, $P(A) \geq 0$.
2. $P(\Omega) = 1$
3. For disjoint sets $A, B \in \mathcal{F}$, $P(A \cup B) = P(A) + P(B)$.

Theorem 1

For all $A \in \mathcal{F}$, $P(\overline{A}) = 1 - P(A)$.

Theorem 2

For all $A, B \in \mathcal{F}$, *if* $A \subseteq B$, *then* $P(A) \leq P(B)$.

Theorem 3

For all $A, B \in \mathcal{F}$, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Suspect the following:

- A and B are mutually exclusive (that is, disjoint) events in $\mathcal{F}$.
- C is another event in $\mathcal{F}$.
- $A \cup B \cup C = \mathcal{F}$.
- $P(A) = .4$.
- $P(B) = .2$.

What is $P(A \cup B)$?

Is it possible that $P(C) = .2$?

Adapted from Epp, pg 374

- When rolling a pair of standard dice, what is the probability that the role sums to 7?
- Suppose an urn has 6 white balls and 5 black balls. When drawing 3 balls at once, what is the probability that one is white and the other two black? Adapted from Ross, pg 37
- In a group of n people, what is the probability that at least 2 people have the same birthday? (Assume each day of the year has equal probability of being a person's birthday, except Feb 29, which has probability 0.)

For next time:

Do the exercises on the back of the handout (to turn in at class time).
Take quiz