Discrete probability unit

- Sample spaces, events, axioms or probability (last week Wednesday and Friday)
- The sum and product rules; event-composition method (Monday)
- Conditional, joint, and marginal probabilities (Friday)
- Random variables, introduction (**Today**)
- Random variables, representing distributions (next week Monday)
- Review (next week Wednesday)
- Test (next week Friday)

Today:

- Finishing application of Bayes's theorem
- Motivating examples for random variables
- Random variable definition
- More random variable examples

Formulas and examples mostly from Ross, *A First Course in Probability*, 1997

A game show features an event where a contestant is shown three doors, with the grand prize hidden behind one of them. The contestant picks one door. To raise the drama, the host then opens one of the other doors, showing it to be empty, and gives the contestant the option of switching to the remaining closed door. Strategically, should the contestant switch?

Theorem 1 (Bayes's formula)

For any events A and B over a sample space,

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Or, more generally, for any partition of the sample space $A_0, A_1, \ldots A_{n-1}$,

$$P(A_j \mid B) = \frac{P(B \mid A_j)P(A_j)}{\sum_{i=0}^{n-1} P(B \mid A_i)P(A_i)}$$

- Consider flipping 4 fair coins. What is the probability of flipping 0 heads? 1 head? 2 heads? 3 heads? 4 heads?
- Consider rolling 2 fair dice. What are the probabilities of the rolled values?
- In one version of the card game Golf, a 6-card hand is scored as 0 for a king, 1 for an ace, 2-10 for cards with those numbers, and 10 for a jack or queen. For example, the hand consisting of the queen of hearts, the 7 of hearts, the 10 of diamonds, the 4 of diamonds, the king of clubs, and the ace of clubs has value 32. What are the probabilities of the various hand values?

Given a sample space Ω, a **random variable** is a function whose domain is Ω. Usually random variables are numerically-valued, that is,

$$f : \Omega \to \mathbb{R}$$

Notationally, we use X, Y, etc for random variables and x, y, etc for values that a random variable can take on, and write

$$P(X = x) = P(\{a \in \Omega \mid X(a) = x\})$$

A random variable defines are partition of the sample space based on the results of a function computed from the basic outcomes. A random variable serves as an abstraction for the phenomenon being modeled.

- An urn has 20 balls numbered 1 through 20. Three balls are selected randomly without replacement. What is the probability that at least one ball has number 17 or larger?
- Consider flipping a fair coin until either getting heads or making 5 flips. What is the probability of each number of flips?
- More generally, suppose the coin isn't fair but has probability p of coming up heads, and suppose you flip it until getting heads, with a maximum of n flips. What is the probability of a given number of flips?

For next time:

Do the exercises on the bottom of the handout.