Discrete probability unit

- Sample spaces, events, axioms or probability (last week Wednesday and Friday)
- The sum and product rules; event-composition method (last week Monday)
- Conditional, joint, and marginal probabilities (last week Wednesday)
- Random variables, introduction (last week Friday)
- Random variables, representing distributions (**Today**)
- Review (Wednesday)
- Test (Friday)

Today:

- Finishing random variable examples
- The idea of a distribution
- CDFs and PMFs

Formulas and examples mostly from Ross, *A First Course in Probability*, 1997

Given a sample space Ω, a **random variable** is a function whose domain is Ω. Usually random variables are numerically-valued, that is,

$$f : \Omega \to \mathbb{R}$$

Notationally, we use X, Y, etc for random variables and x, y, etc for values that a random variable can take on, and write

$$P(X = x) = P(\{a \in \Omega \mid X(a) = x\})$$

A random variable defines are partition of the sample space based on the results of a function computed from the basic outcomes. A random variable serves as an abstraction for the phenomenon being modeled.

- An urn has 20 balls numbered 1 through 20. Three balls are selected randomly without replacement. What is the probability that at least one ball has number 17 or larger?
- Consider flipping a fair coin until either getting heads or making 5 flips. What is the probability of each number of flips?
- More generally, suppose the coin isn't fair but has probability p of coming up heads, and suppose you flip it until getting heads, with a maximum of n flips. What is the probability of a given number of flips?

Let X be a random variable for probability function $P : \Omega \to A$, where A is a countable subset of $\mathbb{R}$. Then the **cumulative distribution function (CDF)** of X is $F_X : A \to [0, 1]$,

$$F_X(x) = P(X \leq x)$$

Note the following:

- If $x \leq y$, then $F_X(x) \leq F_X(y)$.
- $P(x < X \leq y) = F_X(y) - F_X(x)$
- $P(X > x) = 1 - F_X(x)$

Moreover, the **probability mass function (PMF)** is $p_X : A \to [0, 1]$,

$$p_X(x) = P(X = x)$$

Imagine a game that involves 2 dice but, instead of summing the values of the dice, the game makes us of the product of their values. Let X be the random variable representing that product. Compute the probability mass function and cumulative distribution function.

For next time:

Take the quiz on canvas.