Chapter 3, Case Studies:

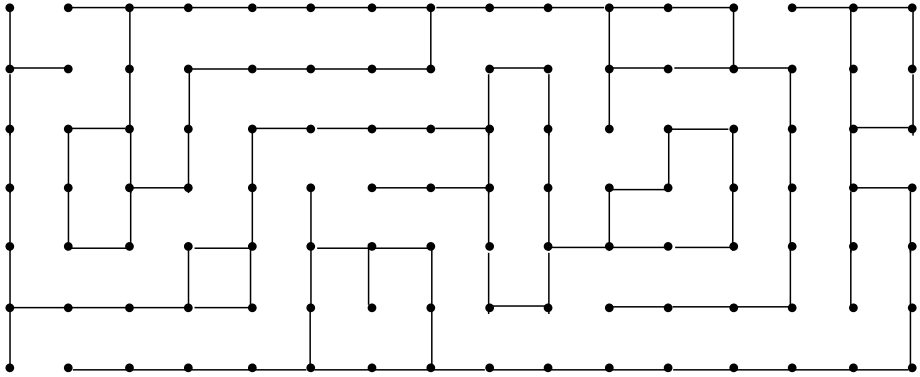
- Linear-time sorting algorithms (Monday and Wednesday)
- Disjoint sets and array forests (**Today**)
- Priority queues and heaps (next week Monday and Wednesday)
- N-sets and bit vectors (next week Thursday lab)
- (Begin Graph unit in lab next week Friday)

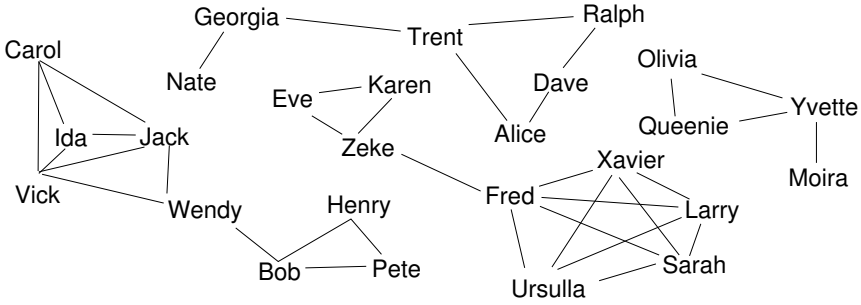
Today:

- Problem statement
- Disjoint set ADT details
- The array forest abstraction and data structure
- Find and union strategies, with optimizations

Problem statement:

> *Suppose we have a collection of items connected by an unknown equivalence relation. Efficiently find the equivalence classes in this collection as information about the relation is discovered.*

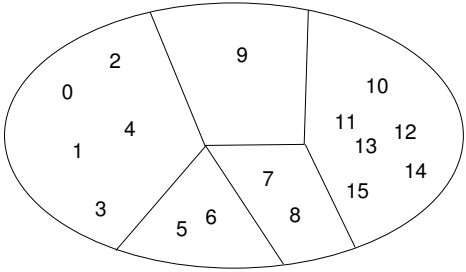


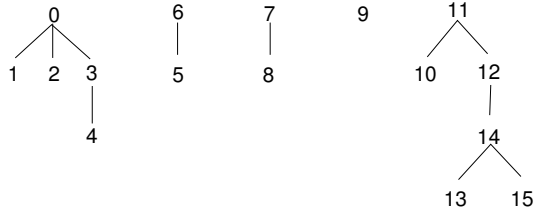

Georgia
Ralph
Trent
Carol
Nate
Dave
Olivia
Eve
Karen
Yvette
Queenie
Ida
Jack
Alice
Zeke
Xavier
Moira
Vick
Fred
Henry
Larry
Wendy
Bob
Pete
Sarah
Ursulla

```
a = c
e = a + b
d = b
g = 1
f = d + c
h = e * g
```

The *disjoint set* ADT:

- Main operations: union two sets, find a set for a given element, and test if two elements are in the same set.
- The universe is closed.
- We assume all elements can be indexed, $[0, N)$.
- A set in the partition is identified by a leader.


2
9
0
10
4
11
12
13
1
14
7
15
3
6
8
5


0
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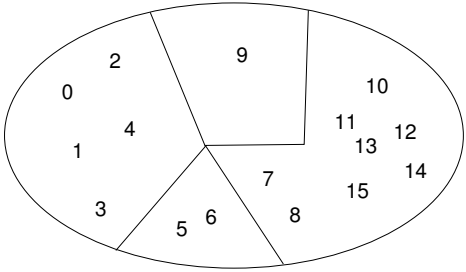

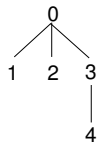

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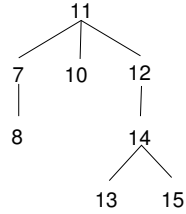


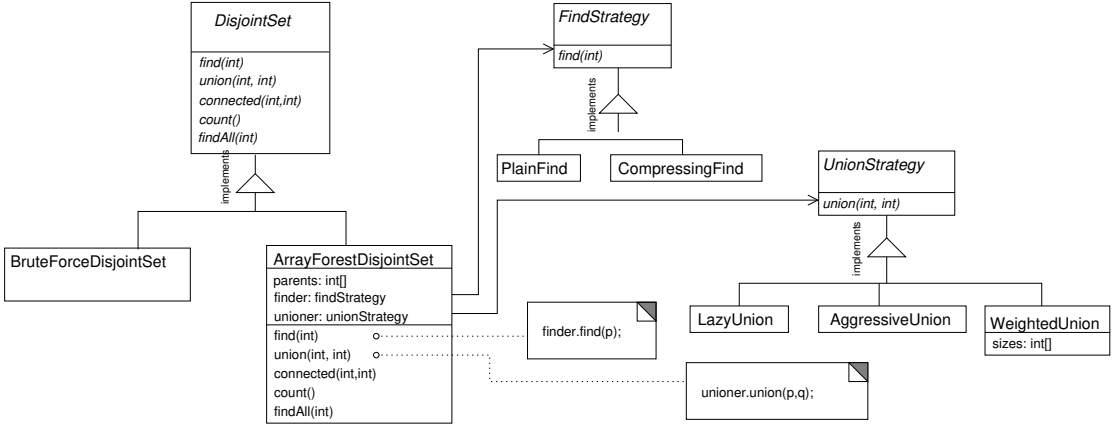
Invariant (Class `ArrayForestDisjointSet`)

For all $i \in [0, n)$,

(a) $\mathit{leader}(i) = \mathit{leader}(\texttt{parents}[i])$, *that is,* $\texttt{parents}[i]$ *points to another element in the same set as* i.

(b) $\mathit{leader}(i) = \texttt{parents}[\mathit{leader}(i)]$, *that is, leaders all point to themselves.*

(c) *Following a finite number links implied by* `parents` *will converge, that is, there is no circularity in the tree.*


DisjointSet
find(int)
union(int, int)
connected(int,int)
count()
findAll(int)
implements
BruteForceDisjointSet
ArrayForestDisjointSet
parents: int[]
finder: findStrategy
unioner: unionStrategy
find(int)
union(int, int)
connected(int,int)
count()
findAll(int)
FindStrategy
find(int)
implements
PlainFind
CompressingFind
finder.find(p);
UnionStrategy
union(int, int)
implements
LazyUnion
AggressiveUnion
WeightedUnion
sizes: int[]
unioner.union(p,q);

Union strategy	LazyUnion	AggressiveUnion	WeightedUnion	LazyUnion	WeightedUnion
Find strategy	PlainFind	PlainFind	PlainFind	CompressingFind	CompressingFind
Find heavy:	1.30E7	3.34E7	7.40E5	9.26E5	6.68E5
	(5.68E6)	(8.40E3)	(1.80E4)	(2.38E4)	(9.34E3)
Even mix:	9.89E7	4.41E7	1.20E6	1.56E6	9.80E5
	(1.22E7)	(9.93E3)	(1.97E4)	(2.12E4)	(9.96E3)
Union heavy:	1.62E8	4.39E7	1.40E6	1.71E6	1.04E6
	(1.26E7)	(9.99E3)	(2.01E4)	(1.59E4)	(1.00E4)

Coming up: (all end-of-day)

Do **linear sorting** *project (Mon, Feb 10)*

Due **Today**:
Finish reading Section 3.2 (disjoint sets and array forests)
Do Ex 2.(12 & 16) and 3.8 Take disjoint-sets quiz

Due **Wed, Feb 12**:
Read Section 3.3 (heaps and priority queues)
Take heap/pq quiz

Due **Thurs, Feb 13**:
Read Section 3.4
Do Exercises 3.(26 & 27).
Take N-sets quiz