Chapter 6, Hash tables:

- General introduction; separate chaining (last week Wednesday)
- Open addressing (last week Friday)
- Hash functions (**Today**)
- Perfect hashing (next week Monday)
- Hash table performance (next week Wednesday)

Today:

- Finish open-addressing deletion
- Hash function properties
- Integer hashes
- String hashes
- Experimental results

Hash functions should distribute the keys *uniformly* and *independently*.

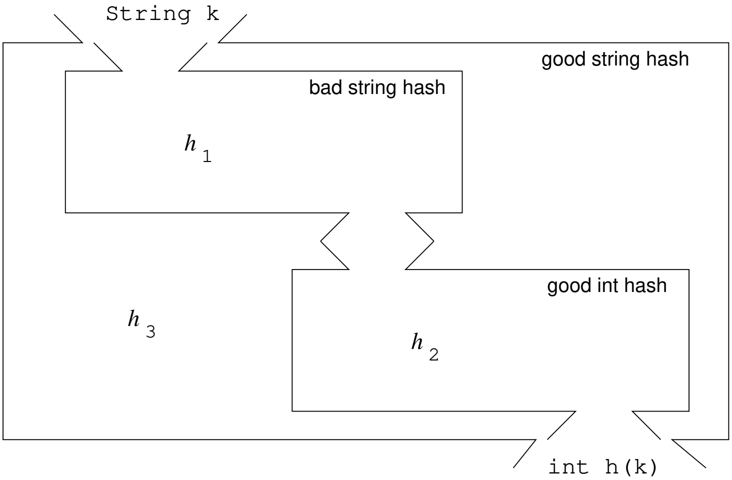
Uniformity:

$$P(h(k) = i) = \frac{1}{m}$$

Independence:

$$P(h(k_1) = i) \quad = \quad P(h(k_1) = i \mid h(k_2) = j)$$

Why do we talk about integer hashes?



Division method:

$$h(k) = k \mod m$$

Middle square method:

	Decimal	Binary
Original	37,914	0000 0000 0000 0000 1001 0100 0001 1010
Squared	1,437,471,396	0101 0101 1010 $\underbrace{\text{1110 0001}}_{\text{middle bits}}$ 0010 1010 0100
Middle 10 bits	225	0000 0000 0000 0000 0000 0000 1110 0001

Multiplicative method:

$$h(k) = \lfloor m(k \cdot a - \lfloor k \cdot a \rfloor) \rfloor$$

"Universal" hash (next time)

ASCII sum:

$$h(k) = \left(\sum_{i=0}^{n-1} s[i]\right)$$

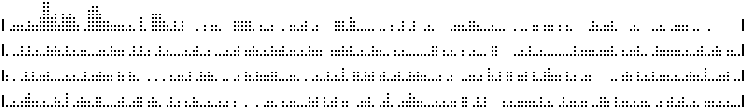
String polynomial:

$$h(k) = (k[0] \cdot b^{n-1} + k[1] \cdot b^{n-2} + \cdots + k[n-2] \cdot b + k[n-1]) \mod m$$

Carter-Wegman:

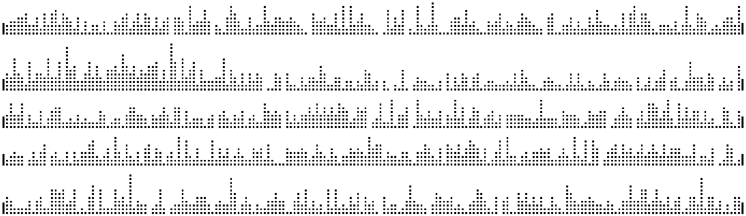
$$\begin{aligned} h(k) &= (h_0(k[0]) + h_1(k[1]) + \cdots + h_{n-1}(k[n-1])) \mod m \\ &= \left(\textstyle\sum_{i=0}^{n-1} h_i(k[i])\right) \mod m \end{aligned}$$

Area codes ($n = 303$)



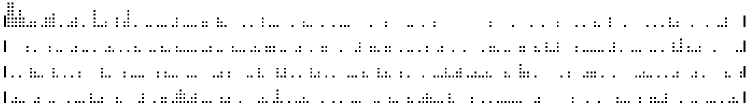
	Average penalty	Variance
Division	.673	.808
Mid square	1.09	1.64
Multiplicative	.508	.478
Fibonacci	.617	.696
Universal	.578	.617

Book ISBNs ($n = 718$)



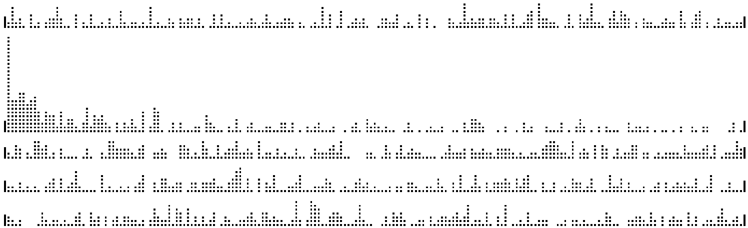
	Average penalty	Variance
Division	.618	1.05
Mid square	.812	1.48
Multiplicative	.565	.954
Fibonacci	.544	.873
Universal	.667	1.15

Randomly generated from $[0, 1000)$ ($n = 150$)



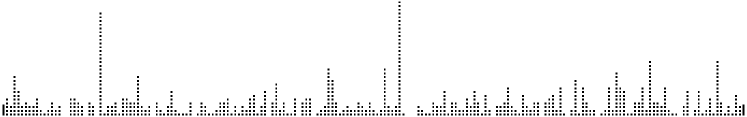
	Average penalty	Variance
Division	1.36	.958
Mid square	1.86	1.96
Multiplicative	1.34	.919
Fibonacci	1.41	1.07
Universal	1.39	1.02

Randomly generated from $[0, 1000)$ ($n = 400$)



	Average penalty	Variance
Division	.518	1.16
Mid square	1.73	3.68
Multiplicative	.405	.930
Fibonacci	.448	.980
Universal	.488	1.08

Chemicals ($n = 663$)		Average penalty	Variance
ASCII sum		.505	1.00
String polynomial		.424	.805
Carter-Wegman		.800	1.63

Books ($n = 718$)			
ASCII sum		.818	1.51
String polynomial		.745	1.30
Carter-Wegman		2.06	4.08

Randomly generated strings ($n = 150$)		Average penalty	Variance
ASCII sum		1.32	.879
String polynomial		1.43	1.09
Carter-Wegman		1.41	1.05

Randomly generated strings ($n = 400$)			
ASCII sum		.515	1.15
String polynomial		.425	.925
Carter-Wegman		.540	1.20

Coming up:

Do **Optimal BST** *project (Due Mon, Nov 25)*
Do **Open addressing with linear probing** *project (due Monday, Dec 2)*

Due **Fri, Nov 22** *(end of day)*
Read Section 7.3
Do Exercises 7.(4,5,7,8)
Take quiz (open addressing) ⟵ *changed duedate to today, Nov 25*

Due **Mon, Dec 2** *(but recommended before break)*
Read Sections 7.(4 & 5)
(No exercises or quiz)

Due **Wed, Dec 4** *(end of day)*
Re-read the last part of Section 7.3
Take quiz (hash table performance)