## Theorem 5.1 (Logical equivalences.) Given logical variables

$p, q$, and $r$, the following equivalences hold.

Commutative laws:
Associative laws:
Distributive Iaws:
Absorption laws:
Idempotent laws:
Double negative law:
DeMorgan's laws:
Negation laws:
Universal bound Iaws:
Identity laws:
Taut/contra laws:

$$
\begin{array}{rlrl}
p \wedge q & \equiv q \wedge p & p \vee q & \equiv q \vee p \\
(p \wedge q) \wedge r & \equiv p \wedge(q \wedge r) & (p \vee q) \vee r & \equiv p \vee(q \vee r) \\
p \wedge(q \vee r) & \equiv(p \wedge q) \vee(p \wedge r) & p \vee(q \wedge r) & \equiv(p \vee q) \wedge(p \vee r) \\
p \wedge(p \vee q) & \equiv p & p \vee(p \wedge q) & \equiv p \\
p \wedge p & \equiv p & p \vee p & \equiv p \\
\sim \sim p & \equiv p & & \\
\sim(p \wedge q) & \equiv \sim p \vee \sim q & \sim(p \vee q) & \equiv \sim p \wedge \sim q \\
p \vee \sim p & \equiv T & p \wedge \sim p & \equiv F \\
p \vee T & \equiv T & p \wedge F & \equiv F \\
p \wedge T & \equiv p & p \vee F & \equiv p \\
\sim T & \equiv F & \sim F & \equiv T
\end{array}
$$

| Is spinach on sale? Do I go to the store? | Is it true that if <br> spinach is on sale, I <br> go to the store? |  |
| :---: | :---: | :--- |
| Yes. | Yes. |  |
| Yes. | No. |  |
| No. | Yes. |  |
| No. | No. |  |

If spinach is not on sale, then I go to the store.

If spinach is not on sale, I do not go to the store.

If spinach is on sale, I do not go to the store.

An even degree is a necessary condition for a polynomial to have no real roots
means
If a polynomial function has no real roots, then it has an even degree.

A positive global minimum is a sufficient condition for a polynomial to have no real roots means
If a polynomial function has a positive global minimum, then it has no real roots.

Values all of the same sign is a necessary and sufficient condition for a polynomial to have no real roots.
means
A polynomial function has values all of the same sign if and only if the function has no real roots.

