Proof. Let the predicate $I(n)$ be "for any set of n cows, every cow in that set has the same color." We will prove $I(n)$ for all $n \geq 1$ by induction on n.

Base case. Suppose we have a set of one cow. Since that cow is the only cow in the set, it obviously has the same color as itself. Thus all the cows in that set have the same color. Hence $I(1)$. Moreover, $\exists N \geq 1$ such that $I(N)$.

Inductive case. Now, suppose we have a set, C, of $N+1$ cows. Pick any cow, $c_1 \in C$. The set $C - \{c_1\}$ has N cows, by Exercise 10 of Chapter 12. Moreoever, by $I(N)$, all cows in the set $C - \{c_1\}$ have the same color. Now pick another cow $c_2 \in C$, where $c_2 \neq c_1$. We know c_2 must exists because $|C| = N + 1 \geq 1 + 1 = 2$. By reasoning similar to that above, all cows in the set $C - \{c_2\}$ must have the same color.

Now, $c_1 \in C - \{c_2\}$, and so c_1 must have the same color as the rest of the set (that is, besides c_2). Similarly, $c_2 \in C - \{c_1\}$, so c_2 must have the same color of the rest of the set. Hence c_1 and c_2 also have the same color as each other, and so all cows of C have the same color.

Therefore, by math induction, $I(n)$ for all n, and all cows of any sized set have the same color. $\square$