Exercise 6 *If $a \in A$ then $\mathscr{P}(A - \{a\}) \cap \{\{a\} \cup A' | A' \in \mathscr{P}(A - \{a\})\} = \emptyset$.*

Proof. Suppose $a \in A$.

We have now reduced this problem into something of the form $B = \emptyset$, Set Form 3. This means we're going to have to use proof by contradiction: Suppose something is an element of the left side, and show that that supposition leads to a contradiction.

Further suppose $X \in \mathscr{P}(A - \{a\}) \cap \{\{a\} \cup A' | A' \in \mathscr{P}(A - \{a\})\}$

We chose a capital X to remind us that X is a set. Now, what does this supposition mean?

By the definition of intersection, $X \in \mathscr{P}(A - \{a\})$ and $X \in \{\{a\} \cup A' | A' \in \mathscr{P}(A - \{a\})\}$.

Remember—X is a set. Think about what the definition of powerset means.

By the definition of powerset, $X \subseteq A - \{a\}$.

Also, the second proposition tells us something more about X. Be careful not to reuse A'. A' is a variable ranging over the elements of $\mathscr{P}(A - \{a\})\}$. Pick a different symbol to refer to a specific one.

By set notation, there exists some $X' \in \mathscr{P}(A - \{a\})\}$ such that $X = \{a\} \cup X'$.

*We have analyzed our supposition. Now, think about this intuitively. Why should this lead to a contradiction? The first part implies that a is not an element of X, but the second part implies that a **is** an element of X. We need to show those two things. Let's start with the second of the two.*

Also by set notation, $a \in \{a\}$. Then, by definition of union, $a \in \{a\} \cup X'$. This means $a \in X$ by substitution.

At this point, we could go on to prove that $a \notin X$, but here's a short cut: Remember that we've said $X \subseteq A - \{a\}$. Then...

By the definition of subset, $a \in A - \{a\}$. By the definition of difference $a \notin \{a\}$. Contradiction.

The contradiction is because we already proved $a \in \{a\}$.

Therefore, by definition of the empty set, $\mathscr{P}(A - \{a\}) \cap \{\{a\} \cup A' | A' \in \mathscr{P}(A - \{a\})\} = \emptyset$. □