**4.2.8.**

**Proof (long version).** Suppose $x \in A \times (B - C)$. By definition of Cartesian product, $x = (a, d)$ for some $a \in A$ and $d \in B - C$. By definition of difference, $d \in B$ and $d \notin C$.

By definition of Cartesian product, $(a, d) \in A \times B$. Also by definition of Cartesian product, this time used negatively, $(a, d) \notin A \times C$.

*[That is, we rewrite $d \notin C$. as $\sim (d \in C)$. By generalization, $\sim (d \in C \wedge a \in A)$. By definition of Cartesian product, $\sim ((a, d) \in A \times C)$. This can be rewritten as $(a, d) \notin A \times C$.]*

By definition of difference, $(a, d) \in (A \times B) - (A \times C)$. By substitution, $x \in (A \times B) - (A \times C)$. Therefore, by definition of subset, $A \times (B - C) \subseteq (A \times B) - (A \times C)$. $\square$

**Proof (short version).** Suppose $(a, d) \in A \times (B - C)$. By definition of Cartesian product, $a \in A$ and $d \in B - C$.

By definition of difference, $d \in B$ and $d \notin C$. By definition of Cartesian product, $(a, d) \in A \times B$ and $(a, d) \notin A \times C$.

By definition of difference, $(a, d) \in (A \times B) - (A \times C)$. Therefore, by definition of subset, $A \times (B - C) \subseteq (A \times B) - (A \times C)$. $\square$