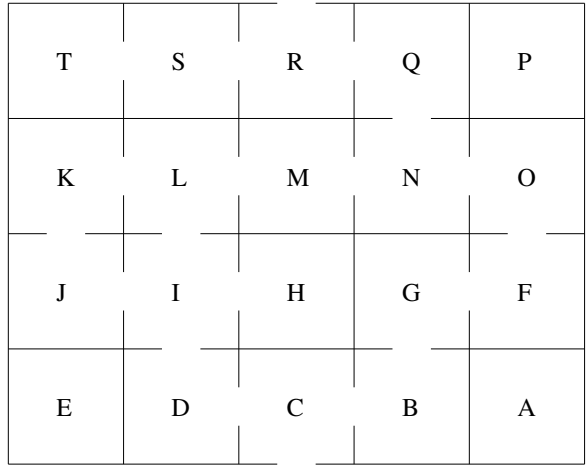
$\forall\ x \in \emptyset, P(x)$ is **always (vacuously) true**.

$\exists\ x \in X \mid P(x)$ is **always false**

$$
\begin{aligned}
&\sim (\forall\, x \in X, P(x)) \\
&\quad \equiv \quad \sim (P(x_1) \wedge P(x_2) \wedge \cdots) \\
&\quad \equiv \quad \sim P(x_1) \vee \sim P(x_2) \vee \cdots \quad \text{By DeMorgan's Law} \\
&\quad \equiv \quad \sim \exists\, x \in X \mid \sim P(x)
\end{aligned}
$$



1. Bob passed through P.
2. Bob passed through N.
3. Bob passed through M.
4. If Bob passed through O, then Bob passed through F.
5. If Bob passed through K, then Bob passed through L.
6. If Bob passed through L, then Bob passed through K.

Based on example by Susanna Epp, 2006