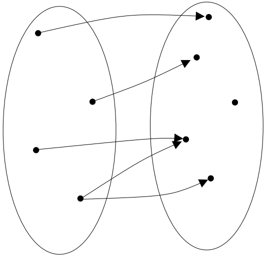
Ex 7.8.8. (From last time) If $f : A \to B$, $g : A \to B$, $h : B \to C$, h is one-to-one, and $h \circ f = h \circ g$, then $f = g$.

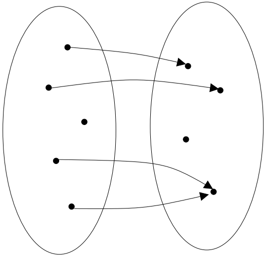
Proof. *Suppose $f : A \to B$, $g : A \to B$, $h : B \to C$, h is one-to-one, and $h \circ f = h \circ g$.*
Suppose $a \in A$. Then

$$\begin{aligned} h(f(a)) &= h \circ f(a) && \text{by definition of function composition} \\ &= h \circ g(a) && \text{by definition of function equality, since } h \circ f = h \circ g \\ &= h(g(a)) && \text{by definition of function composition} \end{aligned}$$

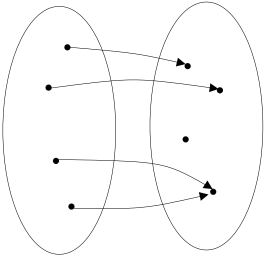
Since $h(f(a)) = h(g(a))$, we then have that $f(a) = g(a)$ by definition of one-to-one (because h is one-to-one). Therefore, by definition of function equality, $f = g$. $\square$



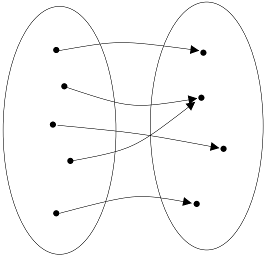
Not a function



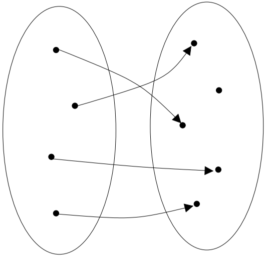
Not a function



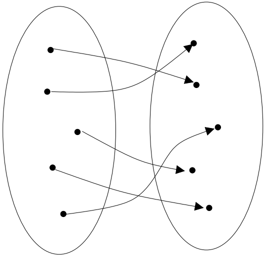
A function but not one-to-one or onto



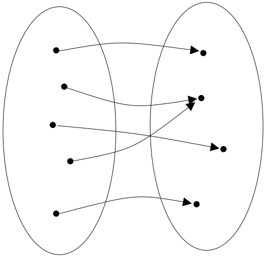
Onto, not one-to-one



One-to-one, not onto

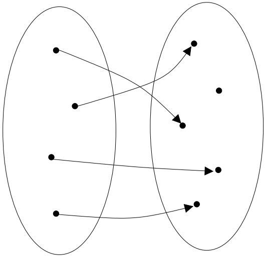


One-to-one correspondence



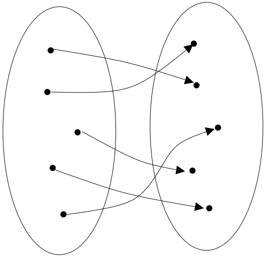
Onto, not one-to-one

$|X| \geq |Y|$



One-to-one, not onto

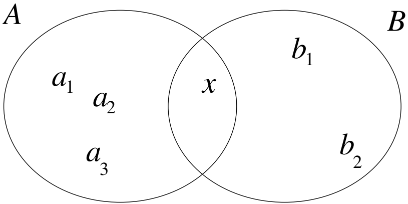
$|X| \leq |Y|$



One-to-one correspondence

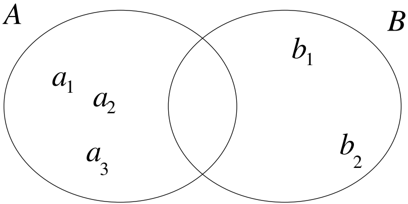
$|X| = |Y|$

$$A \cap B = \emptyset \quad \rightarrow \quad |A \cup B| = |A| + |B|$$



$$|A \cup B| = |\{a_1, a_2, a_3, x, b_1, b_2\} = 6$$

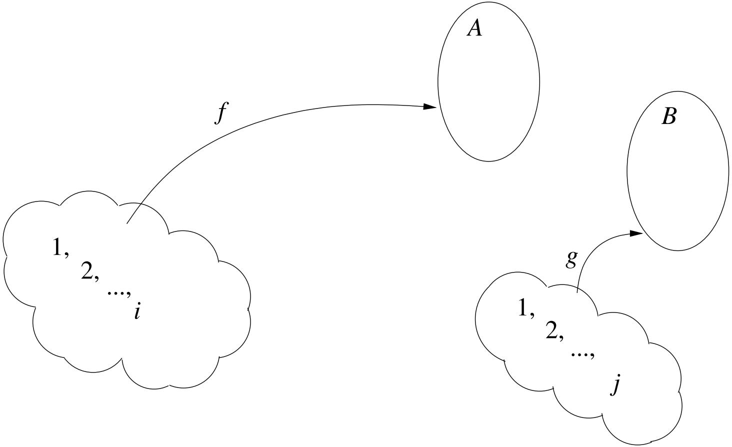
$$|A| + |B| =$$
$$= |\{a_1, a_2, a_3, x\}| + |\{x, b_1, b_2\}|$$
$$= 4 + 3 = 7$$



$$|A \cup B| = |\{a_1, a_2, a_3, b_1, b_2\} = 5$$

$$|A| + |B| =$$
$$= |\{a_1, a_2, a_3\}| + |\{b_1, b_2\}|$$
$$= 3 + 2 = 5$$

$$A \cap B = \emptyset \quad \rightarrow \quad |A \cup B| = |A| + |B|$$



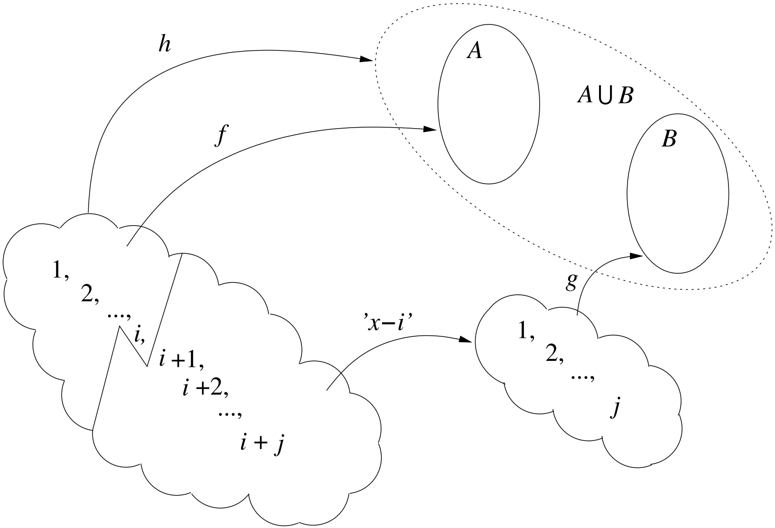
$$A \cap B = \emptyset \quad \rightarrow \quad |A \cup B| = |A| + |B|$$

i	f
1	Zed
2	Yelemis
3	Xavier

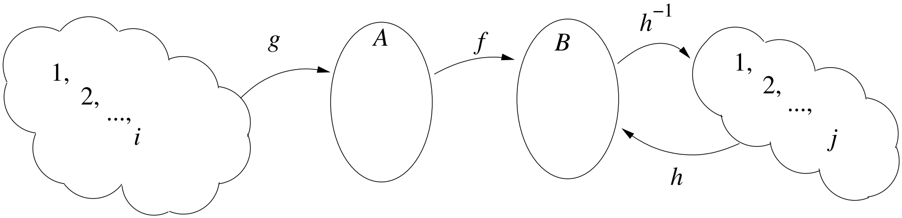
i	g
1	Wilhelmina
2	Valerie
3	Ursula
4	Tassie

i	h				
1	$f(1)$	$=$	Zed		
2	$f(2)$	$=$	Yelemis		
3	$f(3)$	$=$	Xavier		
4	$g(4-3)$	$=$	$g(1)$	$=$	Wilhelmina
5	$g(5-3)$	$=$	$g(2)$	$=$	Valerie
6	$g(6-3)$	$=$	$g(3)$	$=$	Ursula
7	$g(7-3)$	$=$	$g(4)$	$=$	Tassie

$$A \cap B = \emptyset \quad \to \quad |A \cup B| = |A| + |B|$$



$f : A \to B$ is one-to-one $\quad \to \quad |A| \leq |B|$



$f : A \to B$ is one-to-one $\;\to\;$ $|A| \leq |B|$