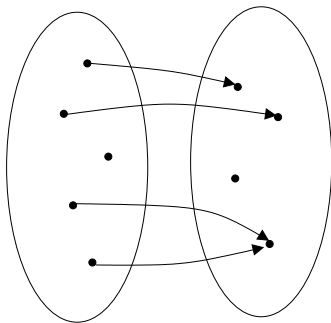


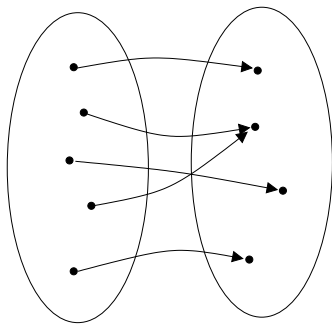
Not a function.

(There's a domain element that is related to two things.)



Not a function.

(There's a domain element that is not related to anything.)



Onto (Surjection)

Everything in the codomain is hit.

$f : X \rightarrow Y$ is onto if $\forall y \in Y,$
 $\exists x \in X \mid f(x) = y.$

Analytic use:

f is onto.

$y \in Y.$

Hence $\exists x \in X$ such that $f(x) = y.$

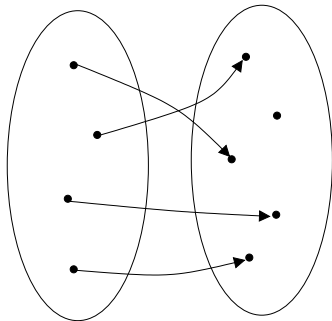
Synthetic use:

Suppose $y \in Y.$

\vdots

(Somehow find x such that $f(x) = y.$)

Therefore f is onto.



One-to-one (Injection)

Domain elements don't share.

f is one-to-one if $\forall x_1, x_2 \in X$,
if $f(x_1) = f(x_2)$ then $x_1 = x_2$.

Analytic use:

f is one-to-one.

$$f(x_1) = f(x_2).$$

Hence $x_1 = x_2$.

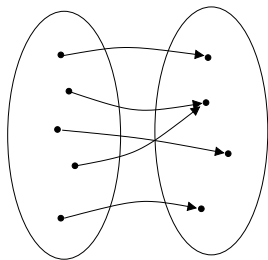
Synthetic use:

Suppose $x_1, x_2 \in X$ and $f(x_1) = f(x_2)$.

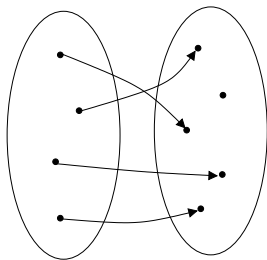
\vdots

(Somehow show $x_1 = x_2$.)

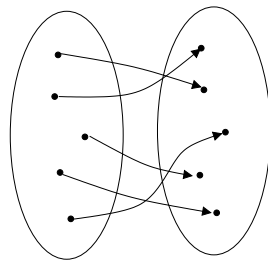
Therefore f is onto.



Onto
(not one-to-one)
 $|X| \geq |Y|$



One-to-one
(not onto)
 $|X| \leq |Y|$



Both onto and one-to-one
 $|X| = |Y|$