Loop invariant: $A[j]$ is the smallest element in the subarray $A[j \ldots n]$. Formally,

(a) $\forall\ k, [j, n], A[j] \leq A[k]$

(b) $n - j$ is the number of iterations completed

Initialization. *Before the loop starts,* $j = n$ *so (a) is trivial and* $j - n = 0$ *implies (b).*

Maintenance. *Suppose the invariant holds at the beginning of an iteration. Let* j_{old} *be the value of* j *before an iteration. Our supposition is, then, that* $\forall\ k, j \leq k \leq n, A[j_{old}] \leq A[k]$ *and* $n - j_{old}$ *is the number of iterations completed.*

Either $A[j_{old} - 1] \leq A[j_{old}]$ *or* $A[j_{old} - 1] > A[j_{old}]$.

Suppose $A[j_{old} - 1] > A[j_{old}]$. *Then by line 4, the values in positions* $j_{old} - 1$ *and* j_{old} *are swapped. Let* A' *be the new state of the array. Then* $A'[j_{old} - 1] < A[j_{old}]$ *and hence* $\forall\ k \in [j_{old} - 1], A[j_{old} - 1] \leq A[k]$.

(continued...) *On the other hand, suppose* $A[j_{old} - 1] \leq A[j_{old}]$*. Then no change is made to the array, so* $A' = A$*. Moreover,* $\forall\ k \in [j_{old} - 1], A[j_{old} - 1] \leq A[k]$*.*

Let j_{new} *be* j *at the end of the iteration. Then* $j_{new} = j_{old} - 1$*, and* $n - j_{old} + 1 = n - j_{new}$ *is the number of iterations completed, satisfying (b). In either case above,* $\forall\ k \in [j_{old} - 1], A[j_{old} - 1] \leq A[k]$*, which satisfies (a).*

Termination. *After* $n - i$ *iterations, by (b)* $n - j = n - i$*, and so* $j = i < i + 1$*. Hence the loop terminates.*

By (a), at termination $\forall\ k, [i, n], A[i] \leq A[k]$*, that is, the value* $A[i]$ *is the smallest value in* A *in the range* $[i, n]$*.* $\square$

Loop invariant: The subarray $A[1, i)$ is sorted and less than everything in subarray $A[i, n]$. Formally:

(a) $\forall\ k \in [1, i), \forall\ \ell \in [i, n], A[k] \leq A[\ell]$
(b) $i - 1$ is the number of iterations completed.

(Note (a) also implies $\forall\ k \in (1, i), A[k-1] \leq A[k]$.)

Initialization. *Before the loop starts,* $i = 1$ *which implies (b). Since the range* $[1, i)$ *is empty, (a) is trivial.*
Maintenance. *Let* i_{old} *and* i_{new} *be the values of* i *before and after the iteration in question. Note that* $i_{new} = i_{old} + 1$ *Suppose that that the invariant holds before the iteration, that is* $i - 1$ *iterations have been completed and*

$$\forall\ k \in [1, i_{old}), \forall\ \ell \in [i_{old}, n], A[k] \leq A[\ell]$$

At the end of the running of the inner loop, $j = i_{old} + 1$, that is, $j = i_{new}$. By the loop invariant we proved in part a,

$$\forall k \in [i_{new}, n], A[i_{new}] \leq A[k]$$

Since all the positions less than i_{new} weren't changed, then this together with our inductive hypothesis tells us that

$$\forall k, [1, i_{new}), \forall \ell \in [i_{new}, n], A[k] \leq A[\ell]$$

Moreover, since $A[i_{new} - 1]$ is greater than everything in positions less than $i_{new} - 1$, we have

$$\forall k \in (1, i_{old}), A[k-1] \leq A[k]$$

Termination. *After $n-1$ iterations, $i-1 = n-1$ by (b), and so $i = n$. Hence the loop terminates*
Moreover, (a) implies that the entire array $A[1 \ldots n]$ is sorted.