

Proof of Theorem 4.5.1 [LP pg 224.]

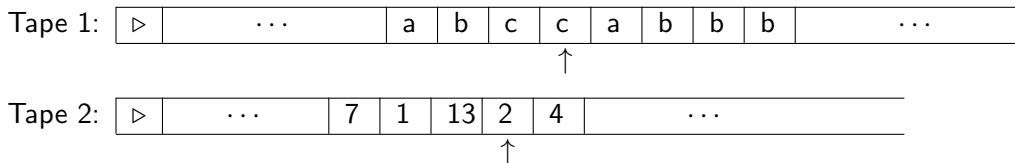
Main idea:

Simulate all computations until you get a halt (if ever).

At a given configuration, say $(q, u\underline{a}v)$. The next step considers only state q and symbol a . The maximum number of next steps is

$$r = |K| \cdot (|\Sigma| + 2)$$

Let M_d be the “deterministic version” of M .



Proof of Theorem 4.5.1, continued.

Define M' with three tapes: One for the original input, one for the tape of the current simulation of M , and one for the current hint tape for M_d . Algorithm for M' :

copy input onto the simulation tape

put 1 onto the hint tape

L: Operate like M_d

if you ever halt, then great!

if you run out of hints,

copy original input back to the simulation tape

put the *lexicographically next* hint on the hint tape

goto L



https://dilbert.com/search_results?terms=Illogical+Scientist

Undecidability has been so much a part of the culture of computer science since its beginnings, that it is easy to forget what a curious fact it is. Strictly speaking, once we accept the identification of problems with languages and algorithms as Turing machines, there are trivial reasons why there must be undecidable problems: There are more languages (uncountably many) than ways of deciding them (Turing machines). Still, that such problems exist so close to our computational ambitions was a complete surprise when it was first observed in the 1930's. Undecidability is in some sense the most lethal form of complexity.

C Papadimitriou, Computational Complexity, pg 59

Gödel's Theorem appears as Proposition VI in his 1931 paper "On Formally Undecidable Propositions in Principia Mathematica and Related Systems I." It states [paraphrased]:

All consistent axiomatic formulations of number theory include undecidable propositions.

Gödel had the insight that a statement of number theory could be about a statement of number theory, if only numbers could somehow stand for statements. The grand conclusion: That the system of [Russel and Whitehead's] Principia Mathematica is "incomplete"—there are true statements of number theory which its methods of proof are too weak to demonstrate.

D Hofstadter, Gödel, Escher, Bach, pg 17–18, abridged

But if Principia Mathematica was the first victim of this stroke, it was certainly not the last. The phrase “and Related Systems” in the title of Gödel’s article is a telling one; for if Gödel’s result had merely pointed out a defect in the work of Russell and Whitehead, then others could have been inspired to improve upon P.M. and to outwit Gödel’s Theorem. But this was not possible: Gödel’s proof pertained to any axiomatic system which purported to achieve the aims which Whitehead and Russell had set for themselves. In short, Gödel showed that provability is a weaker notion than truth, no matter what axiomatic system is involved.

Therefore Gödel’s Theorem had an electrifying affect upon logicians, mathematicians, and philosophers interested in the foundations of mathematics, for it showed that no fixed system, no matter how complicated, could represent the complexity of the whole numbers. Modern readers may not be as non-plussed by this as readers of 1931 were, since in the interim our culture has absorbed Gödel’s Theorem, along with the conceptual revolutions of relativity and quantum mechanics, and their philosophically disorienting messages have reached the public, even if cushioned by several layers of translation (and obfuscation). There is a general mood of expectation, these days, of “limitive” results—but back in 1931, this came as a bolt from the blue.