Chapter 4 roadmap:

- Subset proofs (Wednesday)
- Set equality and emptiness proofs (**Today**)
- Conditional and biconditional proofs (next week Monday)
- Proofs about powersets (new week Wednesday)
- From theorems to algorithms (next week Friday)
- (Start Chapter 5 week after next)

Today:

- Proofs that sets are equal
- Proofs that a set is empty

General forms:

1. Facts (p)
   Set forms
   1. Subset $X \subseteq Y$
   2. Set equality $X = Y$
   3. Set emptiness $X = \emptyset$
2. Conditionals ($p \to q$)
3. Biconditionals ($p \leftrightarrow q$)

$A \times (B - C) \subseteq (A \times B) - (A \times C)$.

Proof (long version). *Suppose* $x \in A \times (B - C)$. *By definition of Cartesian product,* $x = (a, d)$ *for some* $a \in A$ *and* $d \in B - C$. *By definition of difference,* $d \in B$ *and* $d \notin C$.

By definition of Cartesian product, $(a, d) \in A \times B$. *Also by definition of Cartesian product, this time used negatively,* $(a, d) \notin A \times C$.

[That is, we rewrite $d \notin C$. *as* $\sim (d \in C)$. *By generalization,* $\sim (d \in C \wedge a \in A)$. *By definition of Cartesian product,* $\sim ((a, d) \in A \times C)$. *This can be rewritten as* $(a, d) \notin A \times C$.*]*

By definition of difference, $(a, d) \in (A \times B) - (A \times C)$. *By substitution,* $x \in (A \times B) - (A \times C)$. *Therefore, by definition of subset,* $A \times (B - C) \subseteq (A \times B) - (A \times C)$. $\square$

$A \times (B - C) \subseteq (A \times B) - (A \times C)$.

Proof (short version). *Suppose* $(a, d) \in A \times (B - C)$. *By definition of Cartesian product,* $a \in A$ *and* $d \in B - C$.

By definition of difference, $d \in B$ *and* $d \notin C$. *By definition of Cartesian product,* $(a, d) \in A \times B$ *and* $(a, d) \notin A \times C$.

By definition of difference, $(a, d) \in (A \times B) - (A \times C)$. *Therefore, by definition of subset,* $A \times (B - C) \subseteq (A \times B) - (A \times C)$. □

For next time:

Pg 160: 4.3.(3, 14, 15, 18)
Pg 161: 4.4.(5 & 6)

See assignment on Schoology for hint on Ex 4.3.15.

Read 4.(5–8)