Where we are:

- More about functions in SML; introduction to lists (this past Friday)
- Functions on lists; powersets (today)
- Application: A language processor (this coming Friday)
- Propositional forms, logical equivalence [Start Chapter 3] (next week Monday)

Today:

- Functions on lists
  - Examples
  - Principles
  - Practice
- Powersets
  - Definition
  - Exploration

Review

- List literals: `[1, 4, 12, 3]`, `[]`
- Analytic operations: `hd`, `tl`
- Synthetic operations: `::` (cons), `@` (cat)
- Lists vs tuples
- Type analysis problems :

$$\underbrace{[\,\underbrace{\underbrace{\texttt{tl}(\underbrace{[\underbrace{5}_{\text{int}},\underbrace{12}_{\text{int}},\underbrace{6}_{\text{int}}]}_{\text{int list}})}_{\text{int list}} \,\texttt{@}\, \underbrace{[\underbrace{8}_{\text{int}},\underbrace{9}_{\text{int}}]}_{\text{int list}}}_{\text{int list}}\,]}_{\text{int list list}}$$

- Lists as models for sets

Template for functions that process lists

	`repeatEach()`	`sum()`	`switchPairs()`
base case result			
recursive call			
use of head			
combiner			

Powersets

- Informal definition: The powerset of a set is the set of all subsets of that set.
- Formal definition: The powerset of a set X is

$$\mathscr{P}(X) = \{\ Y \mid Y \subseteq X\}$$

- For "set of sets," think "box of boxes."
- Examples:

Why powersets seem to throw some people:

- The elements of a powerset are themselves sets.
- Suppose $X \subseteq \mathcal{U}$. Then
  - If $x \in X$, then $x \in \mathcal{U}$
  - $\mathscr{P}(X) \not\subseteq \mathcal{U}$, but rather $\mathscr{P}(X) \subseteq \mathscr{P}(\mathcal{U})$
  - If $A \in \mathscr{P}(X)$, then $A \in \mathscr{P}(\mathcal{U})$
- $\emptyset \neq \{\emptyset\}$. $|\emptyset| = 0$, but $|\{\emptyset\}| = 1$

Which are true?

$\{3\} \in \mathscr{P}(\{1,2,3,4,5\})$

$3 \in \mathscr{P}(\{1,2,3,4,5\})$

$\{3\} \subseteq \mathscr{P}(\{1,2,3,4,5\})$

$3 \subseteq \mathscr{P}(\{1,2,3,4,5\})$

$a \in A$ iff $\{a\} \in \mathscr{P}(A)$

$a \in A$ iff $\{a\} \subseteq \mathscr{P}(A)$

$a \in A$ iff $\{a\} \subseteq \mathscr{P}(A)$

$A \subseteq B$ iff $A \in \mathscr{P}(B)$

Which are true?

$A \subseteq B$ iff $A \subseteq \mathscr{P}(B)$

$\{A\} \subseteq \mathscr{P}(A)$

$A \in \mathscr{P}(A)$

$\{A\} \in \mathscr{P}(A)$

$\mathbb{Z} \in \mathscr{P}(\mathbb{R})$

$\emptyset = \mathscr{P}(\emptyset)$

Note that

- $a \in A$ iff $\{a\} \in \mathscr{P}(A)$
- $A \subseteq B$ iff $A \in \mathscr{P}(B)$
- $A \subseteq B$ iff $\mathscr{P}(A) \subseteq \mathscr{P}(B)$
- $\mathscr{P}(\emptyset) = \{\emptyset\} \neq \emptyset$

Observe

$$\mathscr{P}(\{1,2,3\}) = \{ \begin{array}{l} \emptyset \\ \{1\},\{2\},\{3\} \\ \{1,2\},\{1,3\},\{2,3\} \\ \{1,2,3\} \end{array} \} = \{ \begin{array}{l} \{1\},\{1,2\},\{1,3\},\{1,2,3\} \\ \emptyset,\{2\},\{3\},\{2,3\} \end{array} \}$$

$$= \mathscr{P}(\{2,3\}) \cup \begin{bmatrix} \text{1 added to each set} \\ \text{of } \mathscr{P}(\{2,3\}) \end{bmatrix} = \mathscr{P}(\{2,3\}) \cup \{ \{1\} \cup X \mid X \in \mathscr{P}(\{2,3\}) \}$$

If $a \in A$, then $\mathscr{P}(A) = \mathscr{P}(A - \{a\}) \cup \{ \{a\} \cup X \mid X \in \mathscr{P}(A - \{a\}) \}$

What is $|\mathscr{P}(X)|$ in terms of $|X|$?

For next time:

If you had trouble on the programming problems from last time, ask for help and try again.

Pg 74: 2.2.(2, 3, 8, 9, 11, 13, 15)

See notes on Ex 2.2.8 and 2.2.9 on the Schoology description of the assignment. See also the code from class for "starter code."

Review 2.4 as necessary
Read carefully 2.5
Skim 2.6

(No quiz for Fri)