

Universal instantiation

$\forall x \in A, P(x)$
 $a \in A$
 $\therefore P(a)$

Universal modus tollens

$\forall x \in A, P(x) \rightarrow Q(x)$
 $a \in A$
 $\sim Q(a)$
 $\therefore \sim P(a)$

Existential instantiation

$\exists x \in A \mid P(x)$
Let $a \in A \mid P(a)$
 $\therefore a \in A \wedge P(a)$

Universal modus ponens

$\forall x \in A, P(x) \rightarrow Q(x)$
 $a \in A$
 $P(a)$
 $\therefore Q(a)$

Existential Generalization

$a \in A$
 $P(a)$
 $\therefore \exists x \in A \mid P(x)$

Hypothetical conditional

Suppose p
 q
 $\therefore p \rightarrow q$

Universal generalization

Suppose $a \in A$
 $P(a)$
 $\therefore \forall x \in A, P(x)$

Hypothetical division into cases

$p \vee q$
Suppose p
 r
Suppose q
 r
 $\therefore r$

(Extra # 2)

(a) $\forall x \in A, P(x)$

(b) $\forall x \in A, x \in B \vee R(x)$

(c) $\forall y \in B, Q(y) \vee \sim P(y)$

(d) $\forall x \in A, R(x) \rightarrow Q(x)$

(e) $\therefore \forall x \in A, Q(x)$

Suppose $a \in A$

(i) $a \in B \wedge R(a)$

Suppose $a \in B$

(ii) $Q(a) \vee \sim P(a)$

(iii) $P(a)$

(iv) $Q(a)$

Suppose $R(a)$

(v) $Q(a)$

(vi) $Q(a)$

(vii) $\therefore \forall x \in A, Q(x)$

by supposition, (b), and UI

by supposition, (c), and UI

by supposition, (a), and UI

by (ii), (iii), and elimination

by supposition, (c), and UMP

by (i), (iv), (v), and HDC

by supposition, (vi), and UG

(Extra # 3)

(a) $\forall x \in A, P(x) \rightarrow R(x)$

(b) $\exists x \in A \mid P(x)$

(c) $\forall x \in A, Q(x) \vee x \in B$

(d) $\forall x \in A, P(x) \rightarrow \sim Q(x)$

(e) $\therefore \exists y \in B \mid R(y)$

Let $a \in A \mid P(a)$

(i) $a \in A \wedge P(a)$

(ii) $a \in A$

(iii) $P(a)$

(iv) $\sim Q(a)$

(v) $Q(a) \vee a \in B$

(vi) $a \in B$

(vii) $R(a)$

(viii) $\therefore \exists y \in B \mid R(y)$

By (b) and EI

By (i) and specialization

By (i) and specialization

by (ii), (iii), (d), and UMP

by (ii), (c), and UI

by (iv), (v), and elimination

by (ii), (iii), (a), and UMP

by (vi), (vii), and EG

3.14.10

	Let $a \in A \mid S(a)$	
1. $\forall x \in A, \exists y \in B \mid P(x, y)$	(i) $a \in A \wedge S(a)$	by (d) and EI
2. $\forall y \in B, Q(y) \vee R(y)$	(ii) $a \in A$	by (i) and specialization
3. $\forall x \in A, y \in B,$ $P(x, y) \rightarrow \sim Q(y)$	(iii) $\exists y \in B \mid P(a, y)$	by (ii), (a), and UI
4. $\exists x \in A \mid S(x)$	Let $b \in B \mid P(a, b)$	
5. $\therefore \exists y \in B \mid R(y)$	(iv) $b \in B \wedge P(a, b)$	by (iii) and EI
	(v) $b \in B$	by (iv) and specialization
	(vi) $P(a, b)$	by (iv) and specialization
	(vii) $\forall y \in B, P(a, y) \rightarrow \sim Q(y)$	by (c), (ii), UI
	(viii) $\sim Q(b)$	by (vii), (v), and UMP
	(ix) $Q(b) \vee R(b)$	by (b), (v), and UI
	(x) $R(b)$	by (ix), (vii), and elimination
	(xi) $\therefore \exists y \in B \mid R(y)$	by (v), (x), and EG

3.14.11

1. $\forall x \in A, x \in B \wedge x \in C$
2. $\forall x \in C, x \in D \vee x \in E$
3. $\forall x \in B, x \in D \rightarrow P(x)$
4. $\forall x \in B, x \in E \rightarrow Q(x)$
5. $\forall x \in B, (P(x) \vee Q(x)) \rightarrow R(x)$
6. $\therefore \forall x \in A, R(x)$

Suppose $a \in A$

(i) $a \in B \wedge a \in C$

by supposition, (a), and UI

(ii) $a \in B$

by (i) and specialization

(ii) $a \in C$

by (i) and specialization

(iv) $a \in D \vee a \in E$

by (iii),(b), and UI

Suppose $a \in D$

(v) $P(a)$

by (ii), supposition, (c), and UMP

(vi) $P(a) \wedge Q(a)$

by (v) and (plain old) generalization

Suppose $b \in E$

(vii) $Q(a)$

by (ii), supposition, (d), and UMP

(viii) $P(a) \vee Q(a)$

by (vii) and (plain old) generalization

(ix) $P(a) \vee Q(a)$

by (iv), (vi), (viii), and HDC

(x) $R(a)$

by supposition, (ix), (e), and UMP

(xi) $\therefore \forall x \in A, R(x)$

by supposition, (x), and UG.

Which of the following are true?

$$-\left((x - y) + (x - z)\right) \equiv -(x - y) - (x - z)$$

$$-\left((x - y) + (x - z)\right) \cdot z \equiv -(x - y) - (x - z) \cdot z$$

$$\sim (p \wedge q) \equiv \sim p \vee \sim q$$

$$\sim (p \wedge q) \wedge r \equiv \sim p \vee \sim q \wedge r$$

Which of the following are true?

$$(x + y) + z = x + (y + z)$$

$$(x - y) + z = x - (y + z)$$

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

$$(p \vee q) \wedge r \equiv p \vee (q \wedge r)$$

1. Write a function `leastSigDigs` that takes a list of ints and returns a list of the least significant digits in those lists. For example, `leastSigDigs[283, 7234, 5, 2380]` would return `[3, 4, 5, 0]`.

2. Write a function `hasEmpty` that takes a list of lists (of any type) and determines whether or not the list of lists contains an empty list. For example, `hasEmpty([[1,2,3], [4,5], [], [6,7]])` would return `true`.