$$\alpha_t(i) = P(\bar{\mathcal{O}}[: t+1], S_t = q_i \mid \lambda) = \begin{cases} \pi_i \cdot b_i(\mathcal{O}_0) & \text{if } t = 0 \\ \left(\sum_{j=0}^{N-1} \alpha_{t-1}(j) \cdot a_{ji} \right) \cdot b_i(\mathcal{O}_t) & \text{otherwise} \end{cases}$$

$$\beta_t(i) = P(\bar{\mathcal{O}}[t+1:] \mid S_t = q_i) = \begin{cases} 1 & \text{if } t = T-1 \\ \\ \displaystyle\sum_{j=0}^{N-1} a_{ij} \cdot b_j(\mathcal{O}_{t+1}) \cdot \beta_{t+1}(j) & \text{if } t < T-1 \end{cases}$$

$$
\begin{aligned}
\delta_t(i) &= \max_{\bar{S}[:t+1]} P(\bar{\mathcal{O}}[:t+1], \bar{S}[:t+1] \mid S_t = q_i) \\
&= \begin{cases} \pi_i \cdot b_i(\mathcal{O}_0) & \text{if } t = 0 \\ \left(\max_{0 \le j < N} \delta_{t-1}(j) \cdot a_{ji} \right) \cdot b_i(\mathcal{O}_t) & \text{otherwise} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\psi_t(i) &= \underset{q_j}{\operatorname{argmax}}\, P(S_{t-1} = q_j, S_t = q_i \mid \bar{\mathcal{O}}[:t+1]) \\
&= \begin{cases} \texttt{None} & \text{if } t = 0 \\ \\ \underset{0 \le j < N}{\operatorname{argmax}}\, \delta_{t-1}(j) \cdot a_{ji} & \text{if } t > 0 \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\xi_t(i,j) &= P(S_t = q_i, S_{t+1} = q_j \mid \bar{\mathcal{O}}, \lambda) \\
&= \frac{P(S_t = q_i, S_{t+1} = q_j, \bar{\mathcal{O}} \mid \lambda)}{P(\bar{\mathcal{O}} \mid \lambda)} \\
&= \frac{\alpha_t(i) \cdot a_{ij} \cdot b_j(\mathcal{O}_{t+1}) \cdot \beta_{t+1}(j)}{\sum_{ii} \sum_{jj} \alpha_t(ii) \cdot a_{ii\ jj} \cdot b_{jj}(\mathcal{O}_{t+1}) \cdot \beta_{t+1}(jj)}
\end{aligned}
$$

$$
\begin{aligned}
\gamma_t(i) &= P(S_t = q_i \mid \bar{\mathcal{O}}, \lambda) \\
&= \sum_{j=0}^{N-1} P(S_t = q_i, S_{t+1} = q_j \mid \bar{\mathcal{O}}, \lambda) \\
&= \sum_{j=0}^{N-1} \xi_t(i,j)
\end{aligned}
$$

$$
\begin{aligned}
\lg \sum_{i=0}^{n-1} x_i &= \lg(x_0 + x_i + \cdots + x_{n-1}) \\
&= \lg x_0 + \lg \left(1 + \sum_{i=1}^{n-1} \frac{x_i}{x_0} \right) \\
&= \lg x_0 + \lg \left(1 + \sum_{i=1}^{n-1} 2^{\lg x_1 - \lg x_0} \right)
\end{aligned}
$$