

## Chapter 4 roadmap:

- ▶ Subset proofs (last week Wednesday)
- ▶ Set equality and emptiness proofs (last week Friday)
- ▶ Conditional and biconditional proofs (**Today**)
- ▶ Proofs about powersets (Wednesday)
- ▶ From theorems to algorithms (Friday)
- ▶ (Start Chapter 5 week after next)

## Today:

- ▶ Proofs of conditional propositions
- ▶ Proofs about numbers
- ▶ Proofs of biconditional propositions

## General forms:

### 1. Facts ( $p$ ) Set forms

1. Subset  $X \subseteq Y$
2. Set equality  $X = Y$
3. Set emptiness  $X = \emptyset$

### 2. Conditionals ( $p \rightarrow q$ )

### 3. Biconditionals ( $p \leftrightarrow q$ )

*Hypothetical conditional* from **Game 3**:

To prove  $p \rightarrow q$

Suppose  $p$

...

$q$

$p \rightarrow q$

An integer  $x$  is *even* if  $\exists k \in \mathbb{Z} \mid x = 2k$ .

An integer  $x$  is *odd* if  $\exists k \in \mathbb{Z} \mid x = 2k + 1$ .

“Axiom 3.” If  $x, y \in \mathbb{Z}$ , then  $x + y \in \mathbb{Z}$ . (*Closure of addition*)

“Axiom 4.” If  $x, y \in \mathbb{Z}$ , then  $x \cdot y \in \mathbb{Z}$ . (*Closure of multiplication*)

“Axiom 5.” If  $x \in \mathbb{Z}$ , then  $x$  is even iff  $x$  is not odd.

$\forall x, y \in \mathbb{Z}$ ,  $x \mid y$  (read, “ $x$  divides  $y$ ”) if  $\exists k \in \mathbb{Z} \mid x \cdot k = y$ .

Note that  $y/x = k$  or  $\frac{y}{x} = k$  or  $x \overline{\mid} \frac{k}{y}$ .

**For next time:**

*Pg 162: 4.5.(1, 4, 5)*

*Pg 164: 4.6.(2 & 5)*

*Pg 165: 4.7.(1 & 6)*

*Review 2.4, especially Ex 2.4.15*

*Skim 4.9*

*Take quiz on Schoology*