Where we are:

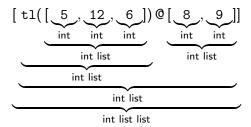
- Making types in SML (last week Wednesday)
- Functions in SML (last week Friday)
- Lists and functions on lists (Wednesday)
- Powersets; a language processor (Today)
- Propositional forms, logical equivalence [Start Chapter 3] (next week Monday)

Today:

- Powersets
 - Definition
 - Exploration
- A language processor
 - Case expressions and option types
 - ► The language processor itself
 - Introducing the semester project

Review

- ► List literals: [1, 4, 12, 3], []
- Analytic operations: hd, t1
- ➤ Synthetic operations: :: (cons), @ (cat)
- Lists vs tuples
- ► Type analysis problems :



Lists as models for sets

Powersets

- ▶ Informal definition: The powerset of a set is the set of all subsets of that set.
- Formal definition: The powerset of a set X is

$$\mathscr{P}(X) = \{ Y \mid Y \subseteq X \}$$

- ► For "set of sets," think "box of boxes."
- Examples:

Why powersets seem to throw some people:

- ▶ The elements of a powerset are themselves sets.
- ▶ Suppose $X \subseteq \mathcal{U}$. Then
 - ▶ If $x \in X$, then $x \in \mathcal{U}$
 - ▶ $\mathscr{P}(X) \not\subseteq \mathcal{U}$, but rather $\mathscr{P}(X) \subseteq \mathscr{P}(\mathcal{U})$
 - ▶ If $A \in \mathcal{P}(X)$, then $A \in \mathcal{P}(\mathcal{U})$
- $ightharpoonup \mathscr{P}(\emptyset) = \{\emptyset\} \neq \emptyset. \ |\emptyset| = 0, \ \mathsf{but} \ |\{\emptyset\}| = 1$

$${3} \in \mathscr{P}({1,2,3,4,5})$$

$$3 \in \mathscr{P}(\{1,2,3,4,5\})$$

$$\{3\}\subseteq\mathscr{P}(\{1,2,3,4,5\})$$

$$3\subseteq\mathscr{P}(\{1,2,3,4,5\})$$

$$a \in A \text{ iff } \{a\} \in \mathscr{P}(A)$$

$$a \in A \text{ iff } \{a\} \subseteq \mathscr{P}(A)$$

$$A \subseteq B$$
 iff $A \subseteq \mathcal{P}(B)$

$$A \subseteq B$$
 iff $A \in \mathscr{P}(B)$

Which are true?

$$A \subseteq B$$
 iff $A \subseteq \mathcal{P}(B)$

$$\{A\}\subseteq\mathscr{P}(A)$$

$$A \in \mathscr{P}(A)$$

$${A} \in \mathscr{P}(A)$$

$$\mathbb{Z}\in\mathscr{P}(\mathbb{R})$$

$$\emptyset = \mathscr{P}(\emptyset)$$

Note that

- ▶ $a \in A \text{ iff } \{a\} \in \mathscr{P}(A)$
- ▶ $A \subseteq B$ iff $A \in \mathcal{P}(B)$
- $ightharpoonup A \subseteq B \text{ iff } \mathscr{P}(A) \subseteq \mathscr{P}(B)$

Observe

If
$$a \in A$$
, then $\mathscr{P}(A) = \mathscr{P}(A - \{a\}) \cup \{\{a\} \cup X \mid X \in \mathscr{P}(A - \{a\})\}$

What is $|\mathscr{P}(X)|$ in terms of |X|?

Grammar:

 $Sentence \rightarrow NounPhrase Predicate PrepPhrase_{opt}$

 $NounPhrase \rightarrow Article\ Adjective_{opt}\ Noun$

 $Predicate \rightarrow Adverb_{opt} VerbPhrase$

Grammar, continued:

 $VerbPhrase
ightarrow \left\{egin{array}{ll} TransitiveVerb \ NounPhrase \\ IntransitiveVerb \\ \\ LinkingVerb \ Adjective \end{array}
ight.$

 $PrepPhrase \rightarrow Preposition NounPhrase$

Vocabulary:

Articles: a the

Adjectives: big bright fast beautiful smart red smelly

Nouns: man woman dog unicorn ball field flea tree

Adverbs: quickly slowly happily dreamily

Transitive verbs: chased saw greeted bit loved

Intransitive verbs: ran slept sang

Linking verbs: was felt seemed

Prepositions: in on through with

For next time:

If you had trouble on the programming problems from last time, ask for help and try again.

Pg 74: 2.2.(11, 13, 15) Pg 82: 2.4.(8-12, 14 & 15)

Read 3.(1-4)