

Chapter 4 roadmap:

- ▶ Subset proofs (last week Wednesday)
- ▶ Set equality and emptiness proofs (last week Friday)
- ▶ Conditional and biconditional proofs (Monday)
- ▶ Proofs about powersets (**Today**)
- ▶ From theorems to algorithms (Friday)
- ▶ (Start Chapter 5 next week)

Today: Case study of large proof (powersets)

- ▶ Review of powersets and their recursive structure
- ▶ Big result
- ▶ Warm-up proofs
- ▶ Proving the big result

Consider the set $A = \{1, 2, 3, 4, 5\}$. Which of the following is true about the powerset $\mathcal{P}(A)$? (Only one is true.)

$$\{3\} \in \mathcal{P}(A)$$

$$3 \in \mathcal{P}(A)$$

$$\{3\} \subseteq \mathcal{P}(A)$$

$$3 \subseteq \mathcal{P}(A)$$

$$A = \{a, b, c\} \qquad \mathcal{P}(A) = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}, \\ \{b, c\}, \{b\}, \{c\}, \emptyset\}$$

$$A - \{a\} = \{b, c\} \qquad \mathcal{P}(A - \{a\}) = \{\{b, c\}, \{b\}, \{c\}, \emptyset\}$$

$$\{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\}$$

$$\mathcal{P}(A) = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}, \\ \{b, c\}, \{b\}, \{c\}, \emptyset\} = \{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} \\ \cup \mathcal{P}(A - \{a\})$$

$$A = \{a, b, c\} \quad \mathcal{P}(A - \{a\}) = \{\{b, c\}, \{b\}, \{c\}, \emptyset\}$$

$$\{\{a\} \cup C \mid C \in \mathcal{P}(A - \{a\})\} = \{\{a, b, c\}, \{a, b\}, \{a, c\}, \{a\}\}$$

If $a \in A$, then $\mathcal{P}(A)$ consists in $\mathcal{P}(A - \{a\})$ and $\{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\}$

Corollary 4.12. If $a \in A$, then $\mathcal{P}(A - \{a\})$ and $\{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\}$ make a partition of $\mathcal{P}(A)$.

$$A \subseteq B \text{ iff } A \in \mathcal{P}(B)$$

$$A \in \mathcal{P}(A)$$

$$\emptyset \in \mathcal{P}(A)$$

$$a \in A \text{ iff } \{a\} \in \mathcal{P}(A)$$

Warm-up proofs:

Theorem 4.7. If $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, then $A \subseteq B$.

Exercise 4.9.1. If $B \subseteq A$, then $\mathcal{P}(B) - \mathcal{P}(A) = \emptyset$.

Roadmap

Corollary 4.12

$\mathcal{P}(A - \{a\})$ and $\{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\}$
make a partition of $\mathcal{P}(A)$.



Theorem 4.11 / Exercise 4.9.6

$$\mathcal{P}(A - \{a\}) \cap \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} = \emptyset$$

Theorem 4.10.

$$\mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} = \mathcal{P}(A)$$



Lemma 4.9.

$$\mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\} \subseteq \mathcal{P}(A)$$

Lemma 4.8.

$$\mathcal{P}(A) \subseteq \mathcal{P}(A - \{a\}) \cup \{C \cup \{a\} \mid C \in \mathcal{P}(A - \{a\})\}$$

For next time:

Pg 174: 4.9.(1, 3, 4, 6)

Skim 4.(10 & 11)