

## Chapter 3 roadmap:

- ▶ Propositions, boolean logic, logical equivalences. **Game 1** (last week Monday)
- ▶ Conditional propositions. **SML** (last week Wednesday)
- ▶ Arguments. **Game 2** (last week Friday)
- ▶ Predicates and quantification. **SML** (Today)
- ▶ Quantified arguments. **Game 3** (Wednesday)
- ▶ Review for test. (Friday)
- ▶ Test 1. (next week Monday)

## Today:

- ▶ Predicates
- ▶ Quantification
- ▶ Practice quantification using programming problems

Project proposal due Friday, Sept 30.

Propositions:

- ▶  $3 < 5$
- ▶ It's Thursday and it is snowing.
- ▶ If  $3 < 5$  then  $12 < 67$ .

Propositional forms:

- ▶  $p \wedge q$
- ▶  $p \rightarrow q$

Four ways to interpret/define the idea of a *predicate*

- ▶ A predicate is a proposition with a parameter.

$$x < 5 \qquad x \text{ is orange}$$

- ▶ A predicate is a function whose value is true or false.

$$P(x) = x < 5 \qquad Q(x) = x \text{ is orange}$$

- ▶ A predicate is a part of a sentence that complements a noun phrase to make a proposition.

A pumpkin **is orange**.

- ▶ A predicate is a truth set

$$\begin{array}{ll} P : \mathbb{N} \rightarrow \mathbb{B}, P(x) = x < 5 & Q(x) = x \text{ is orange} \\ \text{Truth set: } \{1, 2, 3, 4\} & \{ \text{pumpkin, fall leaves, orange juice, } \dots \} \end{array}$$

## Universal quantification

“For all multiples of 3, the sum of their digits is a multiple of 3.”

Let  $D$  be the set of multiples of 3, that is

$$D = \{n \in \mathbb{N} \mid n \bmod 3 = 0\} = \{3, 6, 9, 12, 15, 18, \dots\}$$

$$\forall x \in D, \text{sum}(\text{digify}(x)) \in D$$

Other examples:

- ▶  $\forall x \in \{5, 7, 19, 23, 43\}, x$  is prime.
- ▶  $\forall x \in \{4, 16, 25, 31\}, x$  is a perfect square.

Existential quantification

“There is a multiple of 3 that is not a perfect square.”

$$\exists x \in D \mid x \text{ is not a perfect square}$$

Alternately, “Some multiples of 3 are not perfect squares.”

General forms for universal and existential quantification:

$$\forall x \in X, P(x)$$

$$\exists x \in X \mid P(x)$$

$\forall x \in \emptyset, P(x)$  is **always (vacuously) true**.

$\exists x \in \emptyset \mid P(x)$  is **always false**

$$\sim (\forall x \in X, P(x))$$

$$\equiv \sim (P(x_1) \wedge P(x_2) \wedge \dots)$$

$$\equiv \sim P(x_1) \vee \sim P(x_2) \vee \dots \quad \text{By DeMorgan's Law}$$

$$\equiv \exists x \in X \mid \sim P(x)$$

T	S	R	Q	P
K	L	M	N	O
J	I	H	G	F
E	D	C	B	A

1. Bob passed through *P*.
2. Bob passed through *N*.
3. Bob passed through *M*.
4. If Bob passed through *O*, then Bob passed through *F*.
5. If Bob passed through *K*, then Bob passed through *L*.
6. If Bob passed through *L*, then Bob passed through *K*.

Let  $X$  be the routes through the maze, that is,  
 $X = \{CBGFONQR, CDILMNQR, CDIJKLMNQR\}$

Let  $P(x)$  = route  $x$  contains  $L$ ,  
 $Q(x)$  = route  $x$  contains  $K$ .

Consider  $\forall x \in X, P(x) \rightarrow Q(x)$ .

$X$	$P(x)$	$Q(x)$	$P(x) \rightarrow Q(x)$
$CBGFONQR$			
$CDILMNQR$			
$CDIJKLMNQR$			

T	S	R	Q	P
K	L	M	N	O
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E	D	C	B	A

**For next time:**

*Pg 133: 3.12.(1 & 2)*

*Pg 135: 3.13.(4 & 5)*

*Read 3.14*