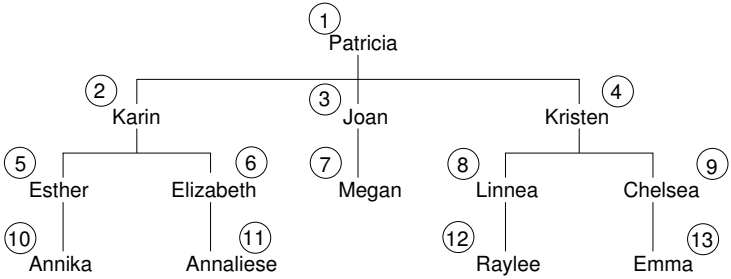
Chapter 3, Case Studies:

- Linear-time sorting algorithms (last week Wednesday and Friday)
- Disjoint sets and array forests (Monday)
- Priority queues (**Today**)
- $N$-sets and bit vectors (Friday )
- (Start graphs Monday)

Today:

- Worklist algorithms
- Priority queue ADT (problem statement)
- Inefficient solutions
- Abstractions for the heap data structure
- Heap implementation details, part 1
- Excursion: heap sort
- Heap implementation details, part 2
- Analysis and optimization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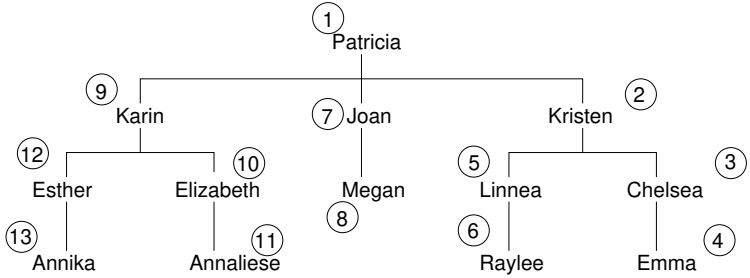
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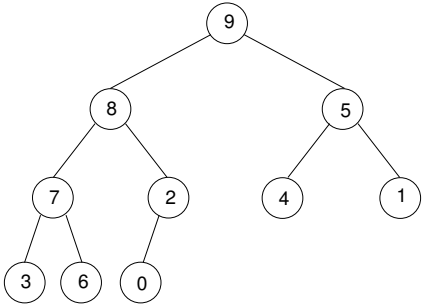
Emma

| | `ListPriorityQueue` | `SortedListPriorityQueue` |
|---|---|---|
| Initialize empty | $\Theta(1)$ | $\Theta(1)$ |
| Initialize populated | $\Theta(n)$ | $\Theta(n^2)$ |
| `insert` | $\Theta(1)$ | $\Theta(n)$ |
| `max` | $\Theta(n)$ | $\Theta(1)$ |
| `extractMax` | $\Theta(n)$ | $\Theta(1)$ |
| `contains` | $\Theta(n)$ | $\Theta(n)$ |
| `increaseKey` | $\Theta(1)$ | $\Theta(n)$ |
| `decreaseKey` | $\Theta(1)$ | $\Theta(n)$ |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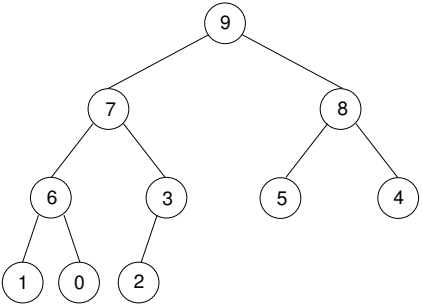
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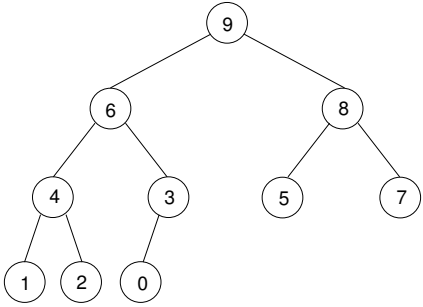
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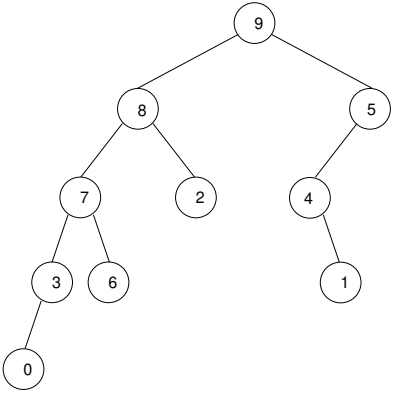
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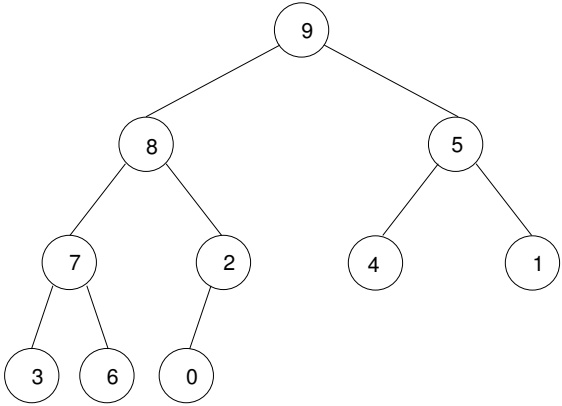


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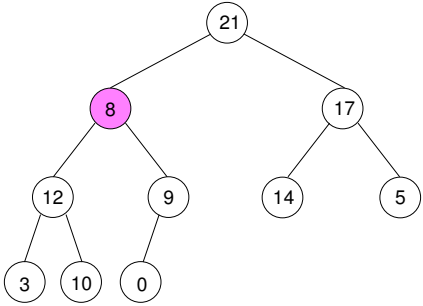
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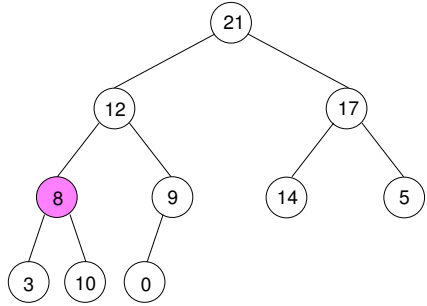
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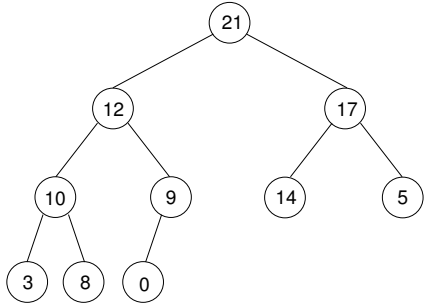
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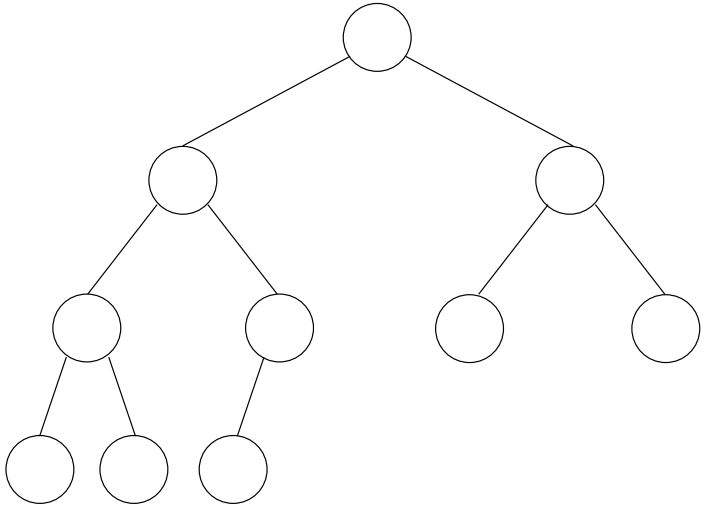
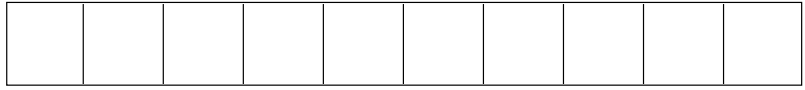
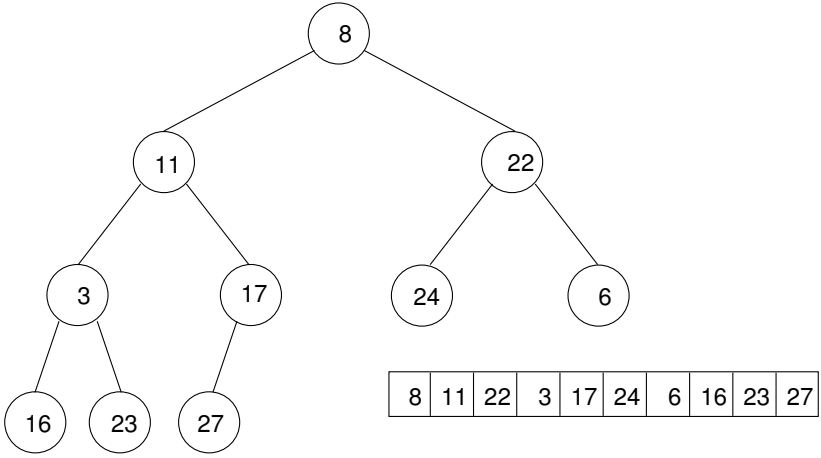




$$
\begin{aligned}
\sum_{i=0}^{h-1} 2^i(h-1-i) &= (h-1)\sum_{i=0}^{h-1} 2^i \;-\; \sum_{i=0}^{h-1} i2^i \\
&= (h-1)(2^h-1) - 2 - (h-2)2^h \\
&= h2^h - 2^h - h + 1 - 2 - h2^h + 2\cdot 2^h \\
&= 2^h - h - 1 \\
&= 2^{\lg(n+1)} - \lg(n+1) - 1 \\
&= n + 1 - \lg(n+1) - 1 \\
&= n - \lg(n+1)
\end{aligned}
$$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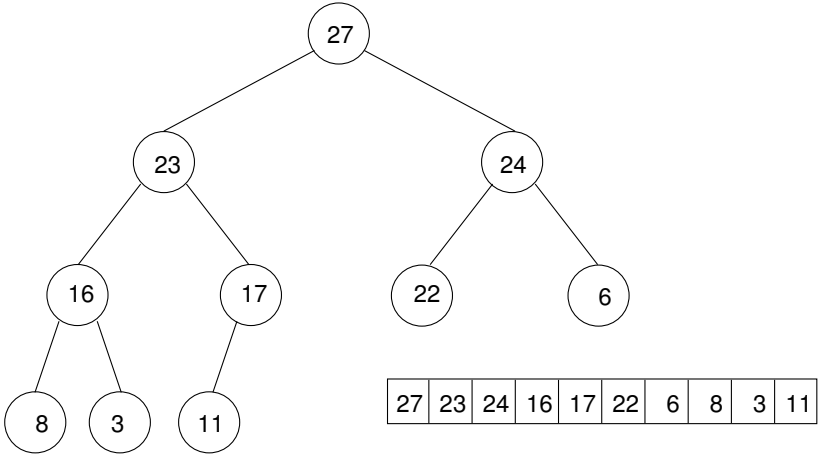


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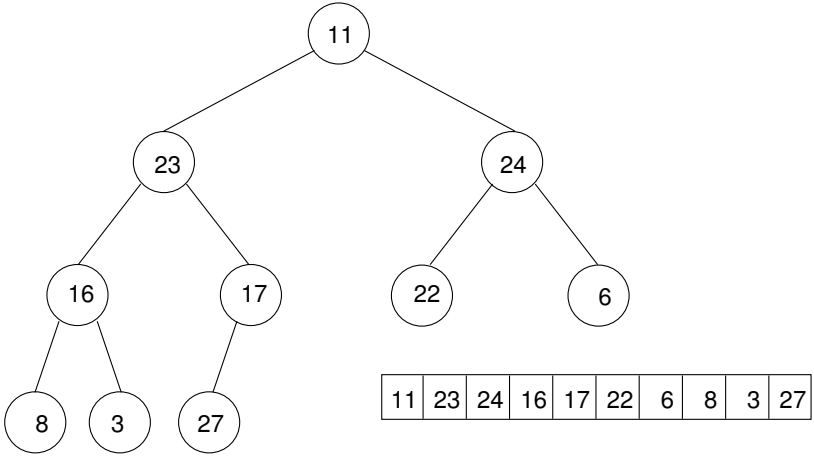
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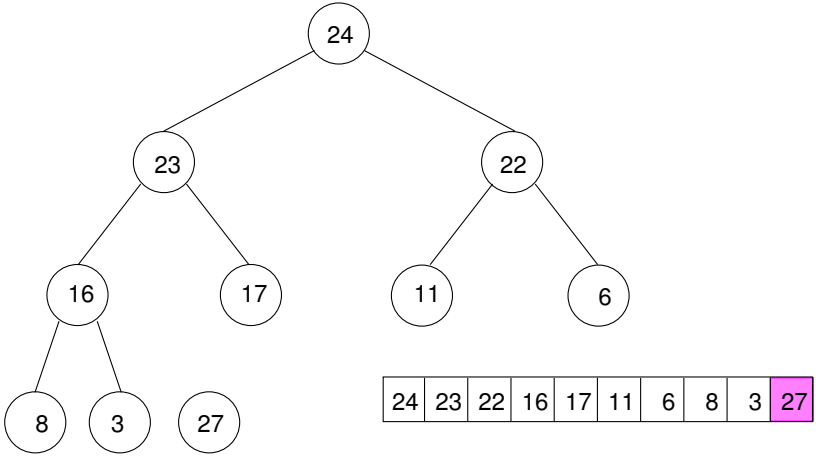
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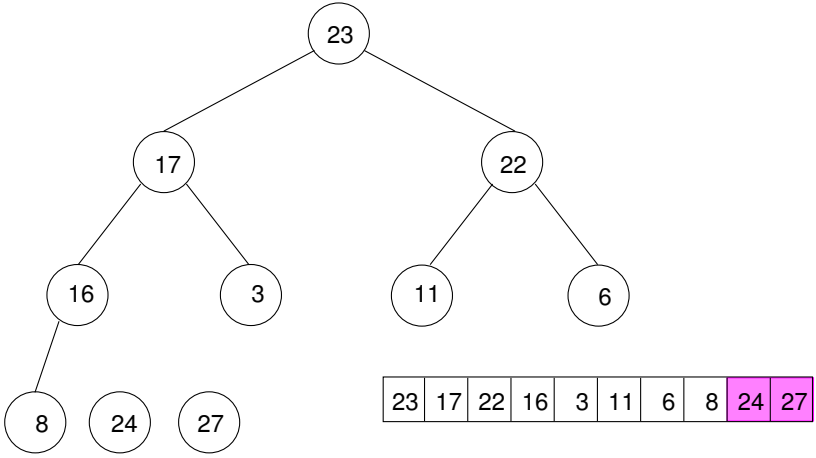
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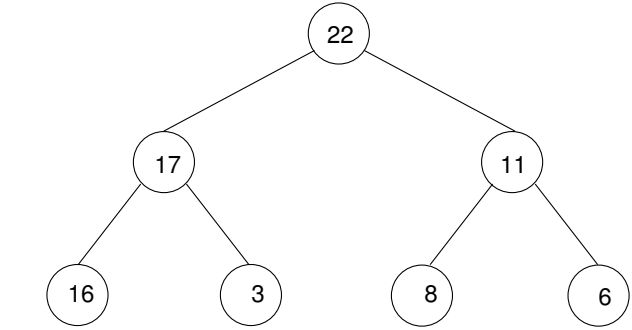
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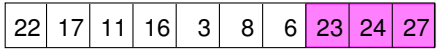
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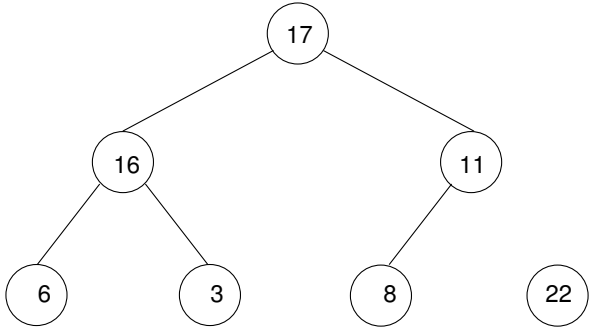
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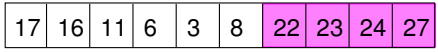
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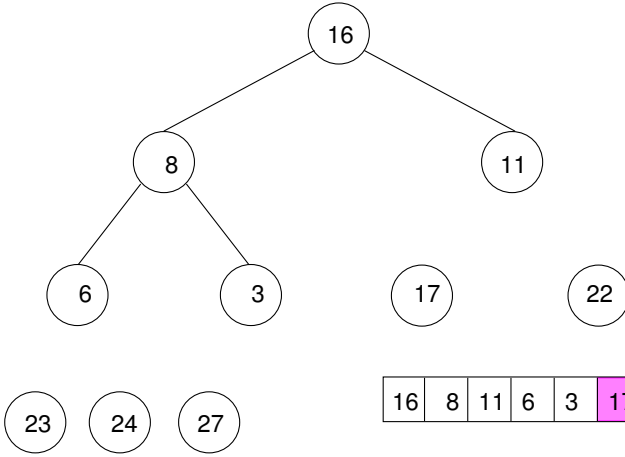


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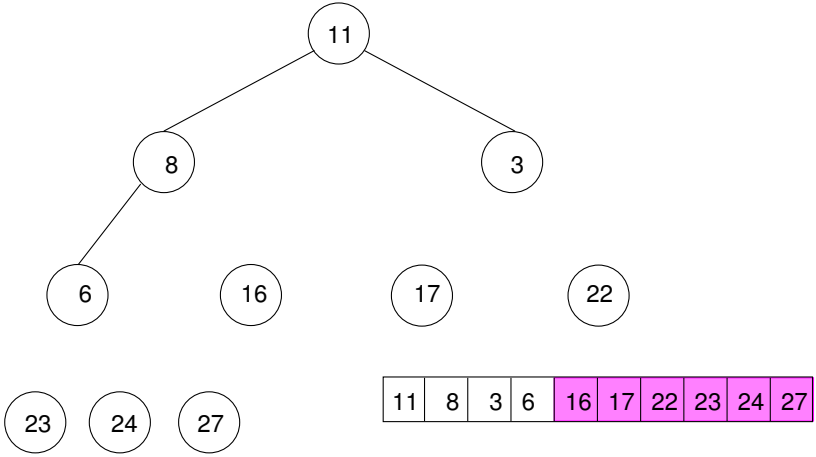
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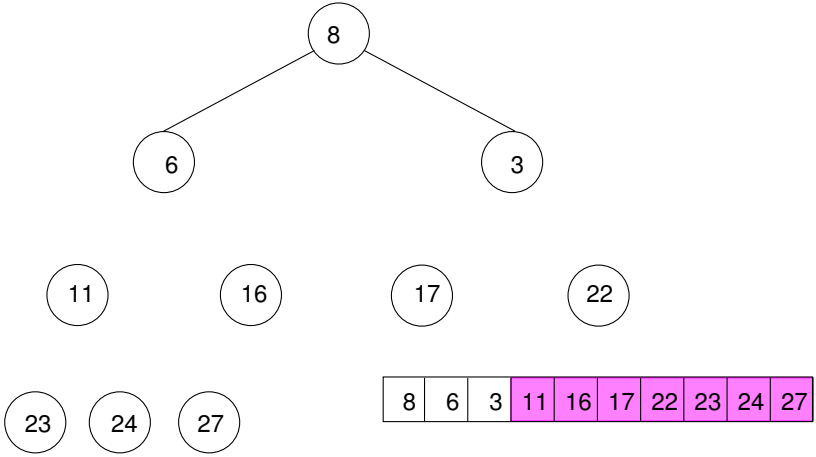
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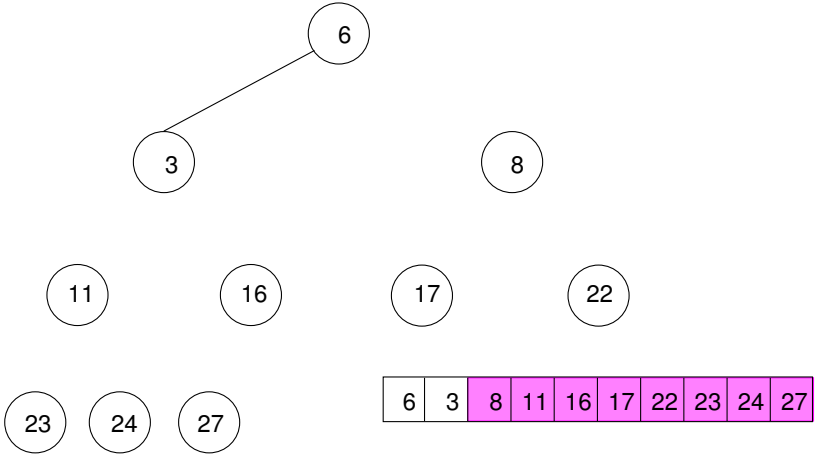
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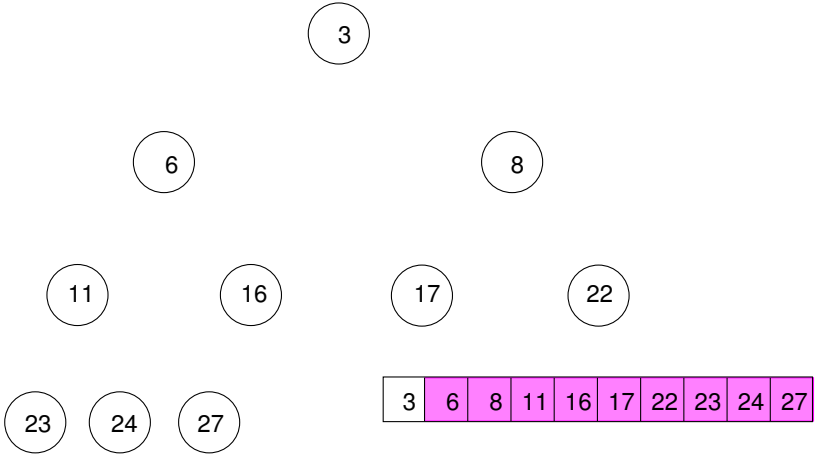
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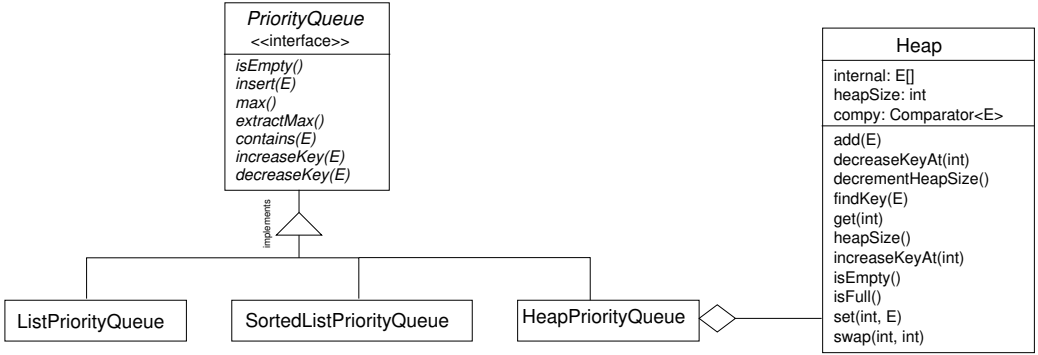
PriorityQueue
<<interface>>
isEmpty()
insert(E)
max()
extractMax()
contains(E)
increaseKey(E)
decreaseKey(E)
implements
Heap
internal: E[]
heapSize: int
compy: Comparator<E>
add(E)
decreaseKeyAt(int)
decrementHeapSize()
findKey(E)
get(int)
heapSize()
increaseKeyAt(int)
isEmpty()
isFull()
set(int, E)
swap(int, int)
ListPriorityQueue
SortedListPriorityQueue
HeapPriorityQueue

| | `ListPriorityQueue` | `SortedPriorityQueue` | `HeapPriorityQueue` |
|---|---|---|---|
| Initialize empty | $\Theta(1)$ | $\Theta(1)$ | $\Theta(1)$ |
| Initialize populated | $\Theta(n)$ | $\Theta(n^2)$ | $\Theta(n)$ |
| `insert` | $\Theta(1)$ | $\Theta(n)$ | $\Theta(\lg n)$ |
| `max` | $\Theta(n)$ | $\Theta(1)$ | $\Theta(1)$ |
| `extractMax` | $\Theta(n)$ | $\Theta(1)$ | $\Theta(\lg n)$ |
| `contains` | $\Theta(n)$ | $\Theta(n)$ | $\Theta(n)$ |
| `increaseKey` | $\Theta(1)$ | $\Theta(n)$ | $\Theta(n)$ |
| `decreaseKey` | $\Theta(1)$ | $\Theta(n)$ | $\Theta(n)$ |

**Coming up:** (all end-of-day)

*Do* **linear sorting** *project (suggested by today)*
*Do* **heaps and priority queue** *project (suggested by Wed, Sept 28)*

*Due* **Today***:*
*Read Section 3.3 (heaps and priority queues)*

*Due* **Thursday***:*
*Take heap/pq quiz*

*Due* **Friday***:*
*Read Section 3.4*
*Do Exercises 3.(26 & 27).*
*Take N-sets quiz*