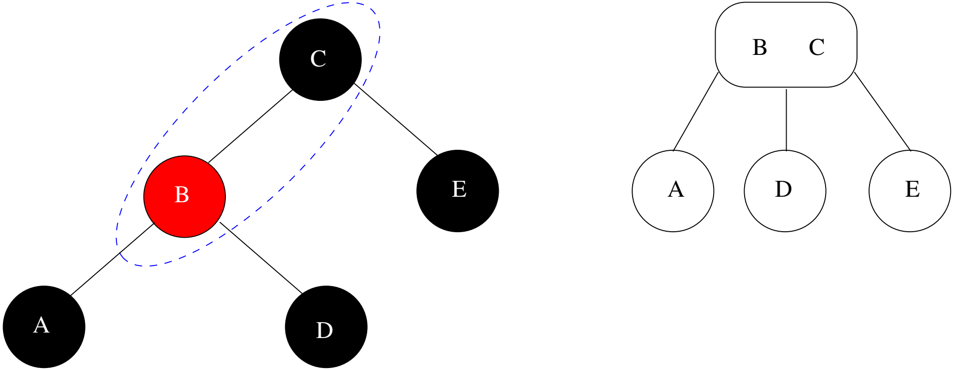
Chapter 5, Binary search trees:

- Binary search trees; the balanced BST problem (Oct 14 & 21)
- AVL trees (Oct 21 & 24)
- Traditional red-black trees (last week Wed, Oct 26)
- Left-leaning red-black trees (last week Fri, Oct 28)
- "Wrap-up" BSTs, B-trees (**Today**)
- Begin dynamic programming (Wednesday)
- Test 2 Friday, Nov 11

Today:

- Finish red-black trees
- B-tree origin stories
  - Two-three trees
  - Sorted arrays
  - Linked/array hybrid
- B-tree definition
- B-tree implementation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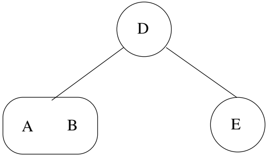
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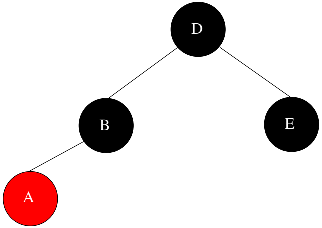
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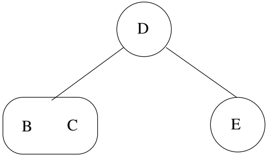
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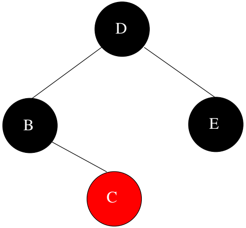
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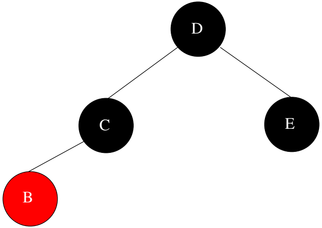
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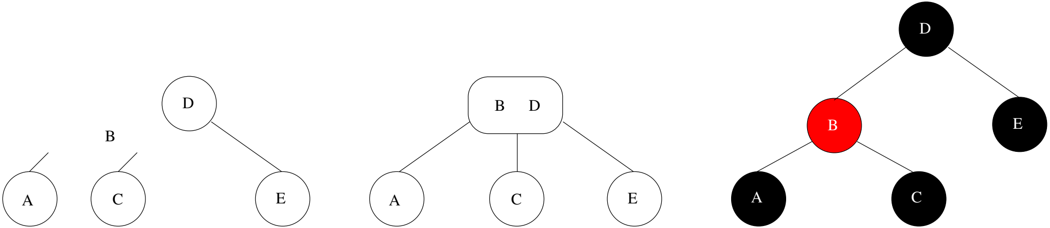
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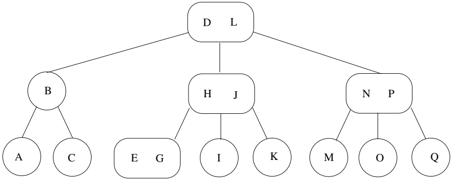
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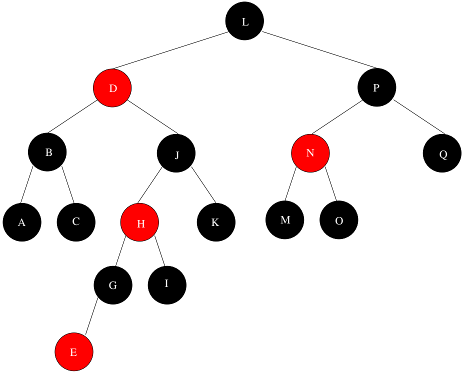
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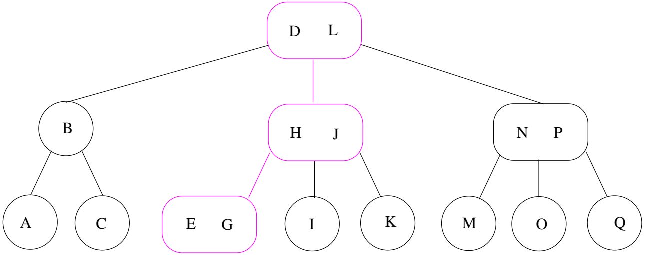
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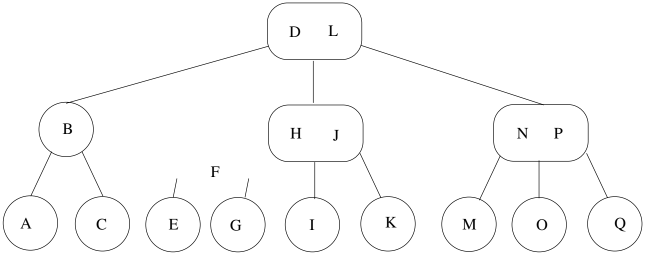
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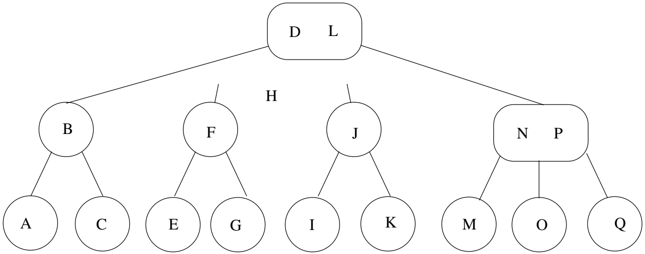
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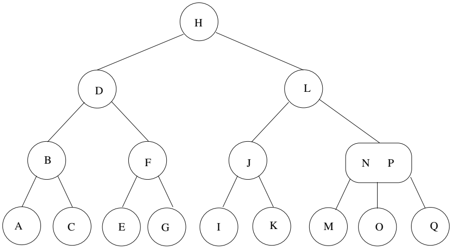
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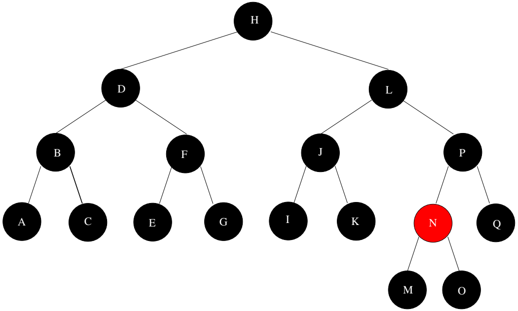
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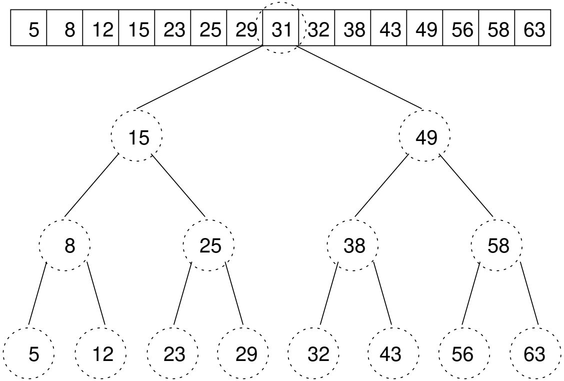
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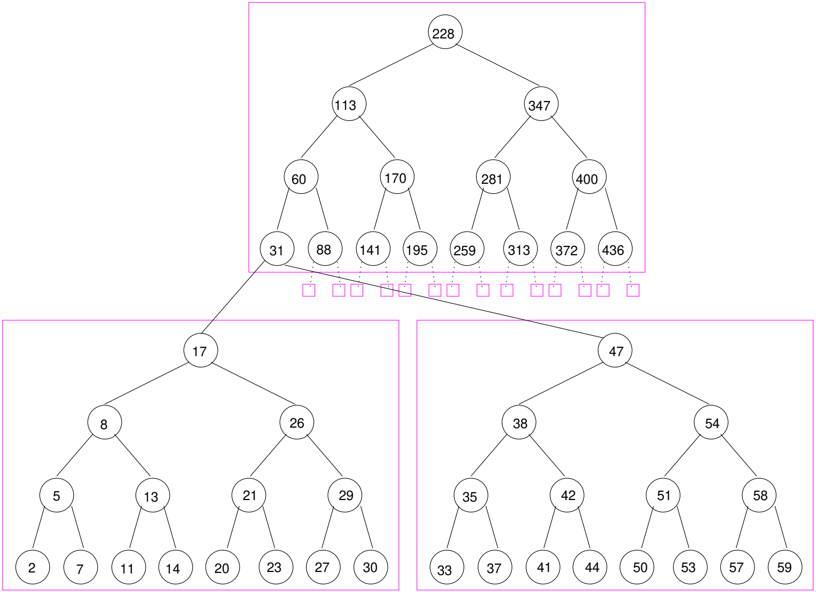
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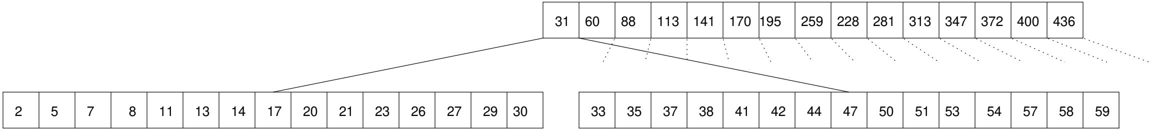
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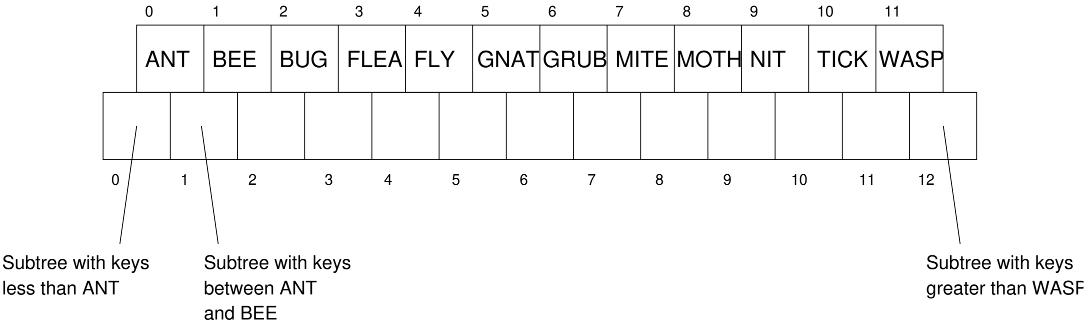
2 5 7 8 11 13 14 17 20 21 23 26 27 29 30
33 35 37 38 41 42 44 47 50 51 53 54 57 58 59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ANT
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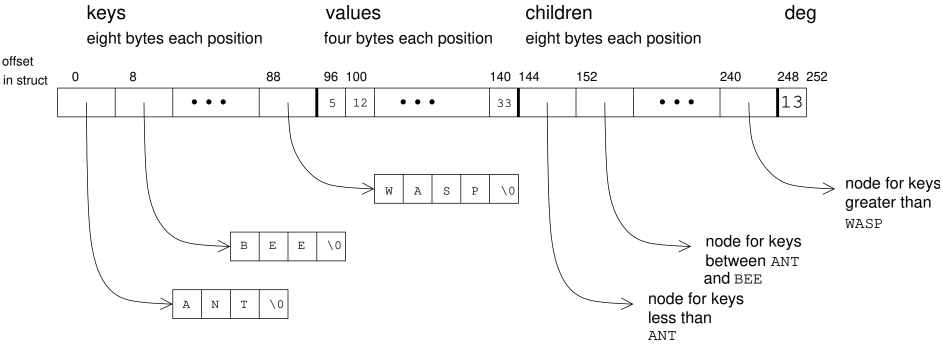
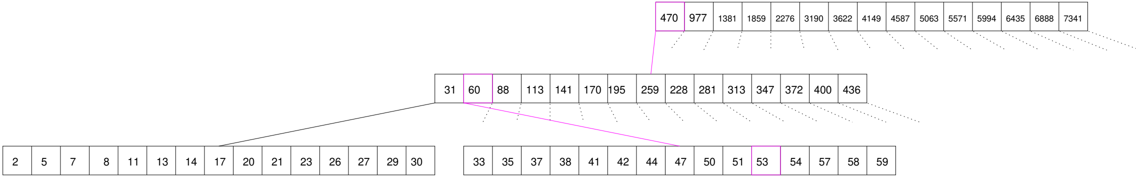
GNAT
GRUB
MITE
MOTH
NIT
TICK
WASP
0
1
2
3
4
5
6
7
8
9
10
11
12
Subtree with keys less than ANT
Subtree with keys between ANT and BEE
Subtree with keys greater than WASP

Formally, a B-tree with maximum degree M over some ordered key type is either

- empty, or
- a node with with $d - 1$ keys and d children, designated as lists `keys` and `children` such that
  - $\lceil M/2 \rceil \leq d \leq M$,
  - `children`[0] is a B-tree such that all of the keys in that tree are less than `keys`[0],
  - for all $i \in [1, d - 1)$, `children`[i] is a B-tree such that all of the keys in that tree are greater than `keys`[$i - 1$] and less than `keys`[i],
  - and `children`[$d - 1$] is a B-tree such that all of the keys in that tree are greater than `keys`[$d - 2$].



keys
eight bytes each position
values
four bytes each position
children
eight bytes each position
deg
offset
in struct
0
8
88
96
100
140
144
152
240
248
252
5
12
33
13
W A S P \0
B E E \0
A N T \0
node for keys greater than WASP
node for keys between ANT and BEE
node for keys less than ANT


470 977 1381 1859 2276 3190 3622 4149 4587 5063 5571 5994 6435 6888 7341
31 60 88 113 141 170 195 259 228 281 313 347 372 400 436
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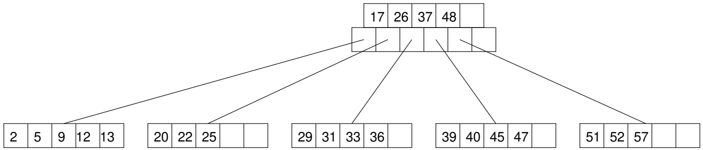
33 35 37 38 41 42 44 47 50 51 53 54 57 58 59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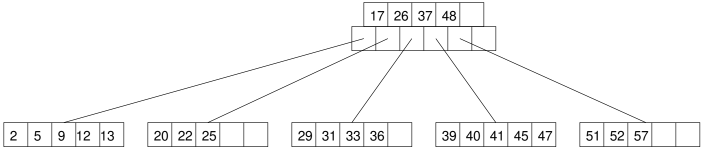
2 5 9 12 13
20 22 25
29 31 33 36
39 40 45 47
51 52 57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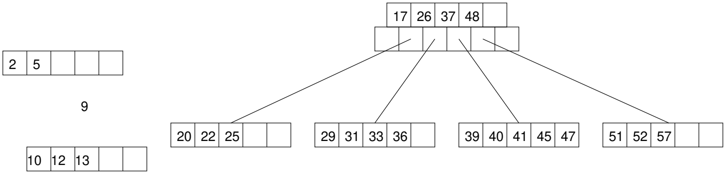
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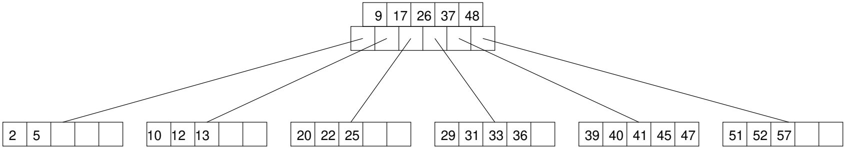
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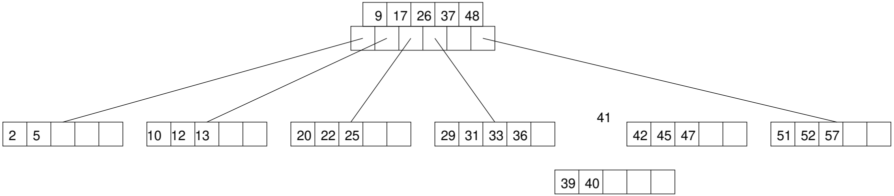
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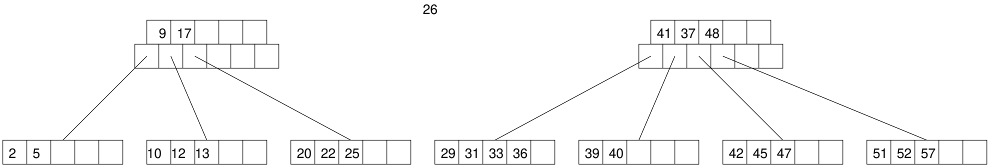
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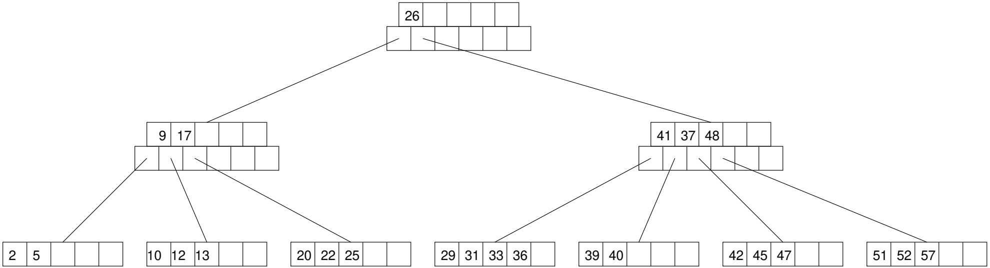
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20 22 25
29 31 33 36
39 40
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$$\underbrace{(M-1)}_{\text{keys per node}} \underbrace{\sum_{i=0}^{h-1} M^i}_{\text{sum of nodes at each level}} = (M-1)\frac{M^h-1}{M-1} = M^h - 1$$

$$\begin{aligned} n &= M^h - 1 \\ M^h &= n + 1 \\ h &= \log_M(n+1) \end{aligned}$$

$$
\begin{aligned}
n &= M^h - 1 \\
M^h &= n + 1 \\
h &= \log_M(n+1)
\end{aligned}
$$

$$
h = \log_{\frac{M}{2}}(n+1) = \frac{\log_M(n+1)}{1 - \log_M 2}
$$

Cost of a search:

$$
\begin{aligned}
\lg M \cdot h &= \lg M \cdot \tfrac{\log_M(n+1)}{1-\log_M 2} \\
&= \lg M \frac{\frac{\lg(n+1)}{\lg M}}{1-\frac{\lg 2}{\lg M}} \\
&= \tfrac{\lg(n+1)}{1-\frac{1}{\lg M}} \\
&= \tfrac{\lg M}{\lg M - 1} \lg(n+1)
\end{aligned}
$$

Compare: $1.44 \lg n$ for AVL trees, $2 \lg n$ for RB trees.

Let c_0 be the cost of searching at a node (proportional to $\lg M$) and c_1 be the cost of reading a node from memory. The the cost of an entire search is

$$(c_0 + c_1)\frac{\log_M(n+1)}{1 - \log_M 2}$$

Now, consolidate the constants by letting $d = \frac{c_0+c_1}{1-\log_M 2}$, and we have

$$d \log_M(n+1)$$

Coming up:

Do **Traditional RB** *project (suggested by Fri, Nov 4)*
(Recommended: Do **Left-leaning RB** *project for your own practice)*

Due **Mon, Oct 31 (today** *(end of day) (but hopefully you've spread it out)*
Read Sections 5.(4-6) [some parts carefully, some parts skim, some parts optional—see Schoology]
Do Exercise 5.14
Take quiz

Due **Thurs, Nov 3** *(end of day)*
Read Section 6.(1&2)
Do Exercises 6.(5–7)
Take quiz

Due **Fri, Nov 4** *(end of day)*
Read Section 6.3
Do Exercises 6.(16, 19, 23, 33)
Take quiz