Formal definitions:

$$\Theta(g(n)) = \{f(n) \mid \exists\ c_1, c_2, n_0 \in \mathbb{R}^+ \text{ such that } \forall\ n \geq n_0, 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)\}$$

$$O(g(n)) = \{f(n) \mid \exists\ c, n_0 \in \mathbb{R}^+ \text{ such that } \forall\ n \geq n_0, 0 \leq f(n) \leq c\ g(n)\}$$

$$\Omega(g(n)) = \{f(n) \mid \exists\ c, n_0 \in \mathbb{R}^+ \text{ such that } \forall\ n \geq n_0, 0 \leq c\ g(n) \leq f(n)\ \}$$

$$o(g(n)) = \{f(n) \mid \forall\ c \in \mathbb{R}^+, \exists\ n_0 \in \mathbb{R}^+ \text{ such that } \forall\ n \geq n_0, 0 \leq f(n) < c\ g(n)\ \}$$

$$\omega(g(n)) = \{f(n) \mid \forall\ c \in \mathbb{R}^+, \exists\ n_0 \in \mathbb{R}^+ \text{ such that } \forall\ n \geq n_0, 0 \leq c\ g(n) < f(n)\ \}$$

If $f(n) = O(g(n))$ and $\lg(g(n)) \geq 1$ and $f(n) \geq 1$ for sufficiently large n, then $\lg(f(n)) = O(\lg(g(n))$.

Scratch work: We need a d such that

$$\begin{aligned}
\lg c + \lg g(n) &\leq d \lg g(n) \\
d &\geq \frac{\lg c}{\lg g(n)} + \frac{\lg g(n)}{\lg g(n)} \\
&\geq \lg c + 1 \\
(\lg c + 1)\lg g(n) &= \lg c \cdot \lg g(n) + \lg g(n)
\end{aligned}$$

Proof. *Suppose $f(n) = O(g(n))$. Then there exist c, n_0 such that for all $n > n_0$, $f(n) \leq c \cdot g(n)$. Then*

$$\begin{aligned}
lg\ f(n) &\leq lg\ c\ g(n) && \text{since lg is increasing} \\
&\leq lg\ c + lg\ g(n) && \text{by log property} \\
&\leq lg\ c \cdot lg\ g(n) + lg\ g(n) && \text{Since } lg\ g(n) \geq 1 \\
&\leq (lg\ c + 1) \cdot lg\ g(n)
\end{aligned}$$

Thus for $n > n_0$, $\lg(f(n)) \leq (lg\ c + 1)\lg(g(n))$.