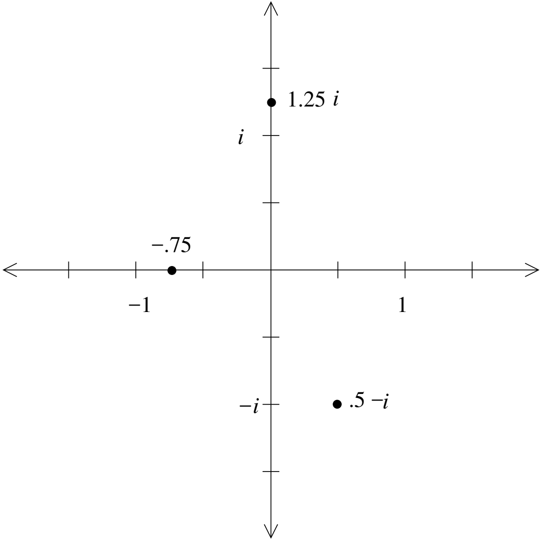


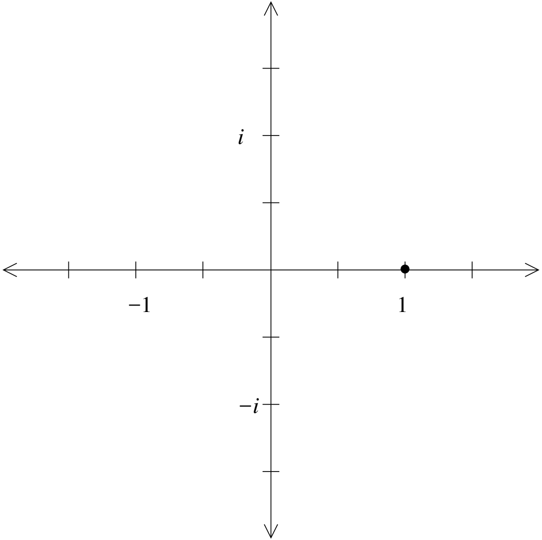
$i = \sqrt{-1}$

$\mathbb{C} = \{a + bi \mid a, b \in \mathbb{R}\}$

$\mathbb{C}$ can be represented as $\mathbb{R} \times \mathbb{R}$.

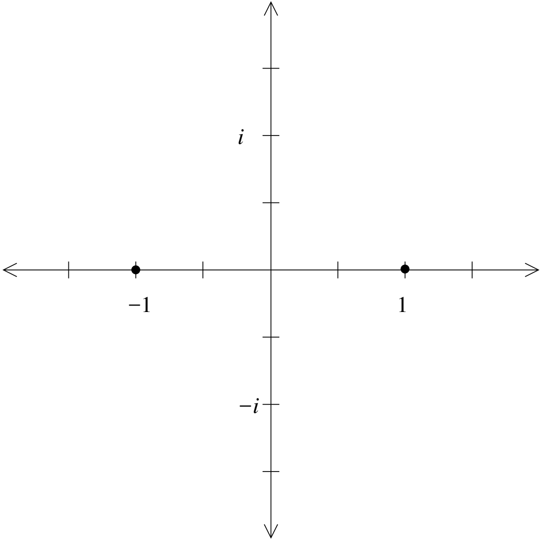
$e^{\pi i} = -1$

An nth *complex root of unity* is $\omega \in \mathbb{C}$ such that $\omega^n = 1$.



$n = 1$

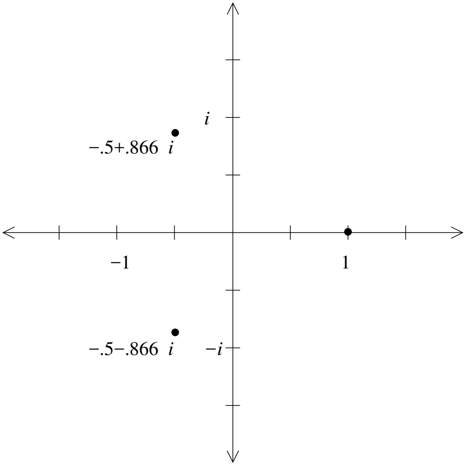
$1^1 = 1$



$n = 2$

$1^2 = 1$

$(-1)^2 = 1$



$n = 3$

$1^3 = 1$

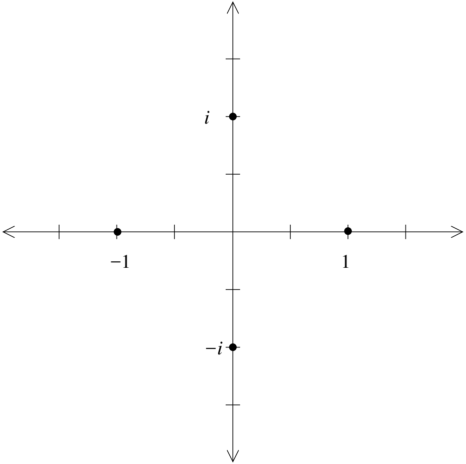
$(e^{\frac{2\pi i}{3}})^3 = (e^{\pi i})^2 = (-1)^2 = 1$

$(e^{\frac{4\pi i}{3}})^3 = (e^{2\pi i})^2 = (1)^2 = 1$

Moreover...

$e^{\frac{2\pi i}{3}} = \cos(\frac{2}{3}\pi) + i\sin(\frac{2}{3}\pi) = -.5 + .866i$

$e^{\frac{4\pi i}{3}} = \cos(\frac{4}{3}\pi) + i\sin(\frac{4}{3}\pi) = -.5 - .866i$



$n = 4$

$1^4 = 1$

$i^4 = (i^2)^2 = (-1)^2 = 1$

$(-1)^4 = 1$

$(-i)^4 = (i^2)^2 = 1$

In general, the *principal nth root of unity* is $\omega_n = e^{\frac{2\pi i}{n}}$

The n complex nth roots of unity are $\omega_n^0, \omega_n^1, \ldots, \omega_n^{n-1}$.

Note that $\omega_n = \omega_n^1$ and $\omega_n^0 = \omega_n^n = 1$.

Note also that $\omega_n^k = e^{\frac{2\pi i}{n}k} = e^{\frac{2k\pi i}{n}} = \cos(\frac{2k\pi}{n}) + i\sin(\frac{2k\pi}{n})$

Cancellation lemma. $\omega_{dn}^{dk} = \omega_n^k$.

Proof. $\omega_{dn}^{dk} = (\omega_{dn})^{dk} = (e^{\frac{2\pi i}{dn}})^{dk} = (e^{\frac{2\pi i}{n}})^k = \omega_n^k$. $\square$

Corollary to above. $\omega_n^{\frac{n}{2}} = \omega_2 = -1$

Proof. Let m be such that $n = 2m$. Then $\omega_n^{\frac{n}{2}} = \omega_{2m}^m = \omega_2 = -1$. $\square$

Cancellation lemma rewritten. If d is a common divisor of n and k, then $\omega_n^k = \omega_{\frac{n}{d}}^{\frac{k}{d}}$.