



```
def linear_search(A, v):  
    i = 0  
    while i < len(A) and A[i] != v :  
        i = i + 1  
    if i == len(A) :  
        return None  
    else  
        return i
```

```

def linear_search(A, v):
    i = 0
    while i < len(A) and A[i] != v :
        i = i + 1
    if i == len(A) :
        return None
    else
        return i

```

- ▶  $\forall k \in [0, i), A[k] \neq v$ .
- ▶  $i$  is the number of iterations completed.

**Init.** Initially,  $i = 0$ , so both parts of the invariant are trivially true.

**Maint.** Suppose that before the iteration,  $\forall k \in [0, i), A[k] \neq v$ , and  $i$  is the number of iterations so far.

In order for the iteration to be executed,  $A[i] \neq v$ . The body of the loop implies  $i_{\text{post}} = i_{\text{pre}} + 1$ . Then  $\forall k \in [1, i_{\text{post}}), A[k] \neq v$ .

Moreover,  $i_{\text{post}}$  is now the number of iterations so far.

(This completes the proof of the lemma that the proposition above *is a loop invariant*.)

```

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    i = 0
    while i < len(A) and A[i] != v :
        i = i + 1
    if i == len(A) :
        return None
    else
        return i

```

- ▶  $\forall k \in [0, i), A[k] \neq v$ .
- ▶  $i$  is the number of iterations completed.

**Term.** By the loop invariant, after  $n$  iterations  $i = n$  and so the guard fails after no more than  $n$  iterations.

When the guard fails, either  $A[i] = v$  or  $i = n$ . In either case, the loop terminates after at most  $n$  iterations.

In the first case,  $A[i] = v$ , and  $i$  is returned. Moreover, by the loop invariant  $i$  is the first position in  $A$  that contains  $v$ .

In the second case  $i = n$  and `None` is returned. By the loop invariant we know that  $\forall k \in [0, n), A[k] \neq v$  and so  $v$  exists nowhere in  $A$ . Either way the algorithm is correct.

```
def linear_search(A, v):  
    found = False  
    i = 0  
    while not found and i < len(A) :  
        found = A[i] == v  
        i = i + 1  
    if found :  
        return i - 1  
    else :  
        return None
```

Invariant:

- ▶  $\forall k \in [0, i - 1), A[k] \neq v$ .
- ▶ found iff  $A[i - 1] = v$
- ▶  $i$  is the number of iterations completed

```
def selection_sort(A):
    for i in range(len(A)) :
        min_pos = i
        min = A[i]
        for j in range(i + 1, len(A)):
            if A[j] < min:
                min = A[j]
                min_pos = j
        A[min_pos] = A[i]
        A[i] = min
```

For next time

*Read Section 2.3*

*Do Ex 2.3-(3, 6, 7)*

*See special instructions for 2.3-7*