Chapter 3 roadmap:

- Propositions, boolean logic, logical equivalences. Game 1 (Monday)
- Conditional propositions. SML (Wednesday)
- Arguments. Game 2 (Today)
- Predicates and quantification. SML (next week Monday)
- Quantified arguments. Game 3 (next week Wednesday)
- Review for test (next week Friday)

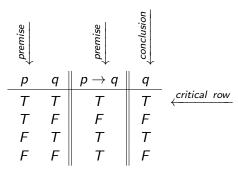
Today:

- Define arguments
- Consider known "syllogisms"
- Practice verifying argument forms (Game 2)

Valid argument

If it is Monday, then it is raining It is Monday.
Therefore it is raining.

$$p \rightarrow q$$
 p
 $\therefore q$



Invalid argument

If it is raining, then there are clouds There are clouds. Therefore it is raining.

$$p \rightarrow q$$
 q
 $\therefore p$

	premise	premise	conclusion	
p	q	$p \rightarrow q$	p	
T	Т	T	T	critical row
T	F	F	T	
F	τ	$\mid \tau \mid$	F	critical row
F	F	T	F	

Alternate definition of validity

Valid argument

p	q	$p \rightarrow q$	$(p \land (p \rightarrow q)) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Invalid argument

p	q	$p \rightarrow q$	$(q \land (p \rightarrow q)) \rightarrow p$
T	T	T	T
T	F	F	T
F	$T \mid$	T	F
F	F	T	T

Modus tollens

If it is spring, then the daffodils bloom. The daffodils aren't blooming. Therefore it is not spring.

p	q	$p \rightarrow q$	$\sim q$	$\sim p$
T	T	T	F	
Τ	F	F	T	
F	T	T	F	
F	F	T	T	T

Modus Ponens

.. q

Modus Tollens

$$p \rightarrow q$$
 $\sim q$

$$\therefore \sim p$$

Generalization

Specialization

$$p \wedge q$$

 $\therefore p$

Elimination

$$p \lor q$$
 $\sim p$

∴ q

Transitivity

$$p \rightarrow q$$

$$q \rightarrow r$$

$$\therefore p \rightarrow r$$

Division into cases

$$p \lor q$$

$$p \rightarrow r$$

$$q \rightarrow r$$

∴. r

Contradiction

$$p \rightarrow F$$

Syllogisms in literature

Elmination:

If anyone knows anything about anything, it's Owl who knows something about something, or my name isn't Winnie-the-Pooh. Which it is. So there you have it.

A. A. Milne, Winnie-the-Pooh, Ch 4.

Division into cases:

Soon her eye fell on a little glass box that was lying under the table: she opened it, and found in it a very small cake, on which the words "EAT ME" were beautifully marked in currants. "Well, I'll eat it," said Alice, "and if it makes me grow larger, I can reach the key; and if it makes me grow smaller, I can creep under the door; so either way I'll get into the garden, and I don't care which happens!"

Lewis Carroll, Alice's Adventures in Wonderland, Ch 1.

Proof by contradiction

$$egin{array}{c|ccccc} p & p
ightarrow F & \sim p \ \hline T & F & F \ \hline F & T & T & \stackrel{critical\ row}{\longleftarrow} \end{array}$$

Restore us to yourself, O LORD, that we may be restored. Renew our days as of old—unless you have utterly rejected us, and you remain exceedingly angry with us.

Lam 5:21–22 (ESV)

Mod Pon
$$p \rightarrow q$$
Mod Tol
 $p \rightarrow q$ p $\sim q$ $\therefore q$ $\sim p$

- 3.9.1
- (a) $t \rightarrow u$
- (b) $p\lor\sim q$
- (c) $p \rightarrow (u \rightarrow r)$
- (d) q
- (e) $:: t \to r$

Contradiction

 $p \rightarrow F$

∴~ p

	$ \text{Mod Tol} \\ p \to q $	Generalizatio	n Specialization $p \wedge q$	Elimination $p \lor q$	Transitivity $p \rightarrow q$	Div into cases $p \lor q$	Contradiction $p \to F$
p	$\sim q$	∴ p ∨ q	∴. <i>p</i>	\sim p	q o r	p o r	∴~ <i>p</i>
∴ q	∴~ <i>p</i>			∴ q	$\therefore p \rightarrow r$	q o r	
						∴ r	

- 3.9.2
- (a) $p \rightarrow t$
- (b) $\sim (q \rightarrow t) \rightarrow w$
- (c) $p \vee q$
- (d) $\sim w$
- (e) ∴ t

- 3.9.8
- (a) w
- (b) $q \rightarrow r$
- (c) $t \rightarrow s$
- (d) $u \rightarrow s$
- (e) $(\sim t \land \sim u) \rightarrow \sim w$
- (f) $(s \lor y) \to (p \to q)$
- (g) $\sim (p \rightarrow r) \vee x$
- (h) ∴ *x*

- 3.9.9
- (a) $p \rightarrow q$
- (b) x
- (c) $\sim (p \vee w) \rightarrow r$
- (d) $q \rightarrow u$
- (e) $x \rightarrow t$
- (f) $w \rightarrow u$
- (g) $r \vee s$
- (h) $r \rightarrow F$
- (i) $:: t \wedge s \wedge u$

- 3.9.10
- (a) $u \rightarrow \sim p$
- (b) $(\sim p \lor q) \to (r \to s)$
- (c) $u \wedge \sim w$
- (d) $t \rightarrow s$
- (e) $(\sim t \land \sim r) \rightarrow w$
- (f) ∴ s

For next time:

Pg 119: 3.8.(3 & 5)

Pg 122: 3.9.(3-7)

Read carefully 3.(10 & 11)

Skim 3.(12 & 13)

Take quiz