Chapter 3 roadmap:

- Propositions, boolean logic, logical equivalences. **Game 1** (Monday)
- Conditional propositions. **SML** (Wednesday)
- Arguments. **Game 2** (Today)
- Predicates and quantification. **SML** (next week Monday)
- Quantified arguments. **Game 3** (next week Wednesday)
- Review for test (next week Friday)

Today:

- Define arguments
- Consider known "syllogisms"
- Practice verifying argument forms (Game 2)

Valid argument

If it is Monday, then it is raining
It is Monday.
Therefore it is raining.

$p \rightarrow q$
p
$\therefore q$

premise ↓		premise ↓	conclusion ↓	
p	q	$p \rightarrow q$	q	
T	T	T	T	← *critical row*
T	F	F	F	
F	T	T	T	
F	F	T	F	

Invalid argument

If it is raining, then there are clouds
There are clouds.
Therefore it is raining.

$p \to q$
q
$\therefore p$

	premise	*premise*	*conclusion*	
p	q	$p \to q$	p	
T	T	T	T	← *critical row*
T	F	F	T	
F	T	T	F	← *critical row*
F	F	T	F	

Alternate definition of validity

Valid argument

p	q	$p \to q$	$(p \wedge (p \to q)) \to q$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Invalid argument

p	q	$p \to q$	$(q \wedge (p \to q)) \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Modus tollens

If it is spring, then the daffodils bloom.
The daffodils aren't blooming.
Therefore it is not spring.

p	q	$p \to q$	$\sim q$	$\sim p$
T	T	T	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	T

Modus Ponens

$p \to q$

p

$\therefore q$

Modus Tollens

$p \to q$

$\sim q$

$\therefore \sim p$

Generalization

p

$\therefore p \vee q$

Specialization

$p \wedge q$

$\therefore p$

Elimination

$p \vee q$

$\sim p$

$\therefore q$

Transitivity

$p \to q$

$q \to r$

$\therefore p \to r$

Division into cases

$p \vee q$

$p \to r$

$q \to r$

$\therefore r$

Contradiction

$p \to F$

$\therefore \sim p$

Syllogisms in literature

Elmination:

> *If anyone knows anything about anything, it's Owl who knows something about something, or my name isn't Winnie-the-Pooh. Which it is. So there you have it.* *A. A. Milne, Winnie-the-Pooh, Ch 4.*

Division into cases:

> *Soon her eye fell on a little glass box that was lying under the table: she opened it, and found in it a very small cake, on which the words "EAT ME" were beautifully marked in currants. "Well, I'll eat it," said Alice, "and if it makes me grow larger, I can reach the key; and if it makes me grow smaller, I can creep under the door; so either way I'll get into the garden, and I don't care which happens!"* *Lewis Carroll, Alice's Adventures in Wonderland, Ch 1.*

Proof by contradiction

$p \to F$
$\therefore \sim p$

p	$p \to F$	$\sim p$	
T	F	F	
F	T	T	← *critical row*

Restore us to yourself, O LORD*, that we may be restored. Renew our days as of old—unless you have utterly rejected us, and you remain exceedingly angry with us.* *Lam 5:21–22 (ESV)*

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.1

(a) $t \to u$

(b) $p \vee \sim q$

(c) $p \to (u \to r)$

(d) q

(e) $\therefore t \to r$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.2

(a) $p \to t$

(b) $\sim (q \to t) \to w$

(c) $p \vee q$

(d) $\sim w$

(e) $\therefore t$

Mod Pon	**Mod Tol**	**Generalization**	**Specialization**	**Elimination**	**Transitivity**	**Div into cases**	**Contradiction**
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.8

(a) w

(b) $q \to r$

(c) $t \to s$

(d) $u \to s$

(e) $(\sim t \wedge \sim u) \to \sim w$

(f) $(s \vee y) \to (p \to q)$

(g) $\sim (p \to r) \vee x$

(h) $\therefore x$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.9

(a) $p \to q$

(b) x

(c) $\sim (p \vee w) \to r$

(d) $q \to u$

(e) $x \to t$

(f) $w \to u$

(g) $r \vee s$

(h) $r \to F$

(i) $\therefore t \wedge s \wedge u$

Mod Pon	Mod Tol	Generalization	Specialization	Elimination	Transitivity	Div into cases	Contradiction
$p \to q$	$p \to q$	p	$p \wedge q$	$p \vee q$	$p \to q$	$p \vee q$	$p \to F$
p	$\sim q$	$\therefore p \vee q$	$\therefore p$	$\sim p$	$q \to r$	$p \to r$	$\therefore \sim p$
$\therefore q$	$\therefore \sim p$			$\therefore q$	$\therefore p \to r$	$q \to r$	
						$\therefore r$	

3.9.10

(a) $u \to \sim p$

(b) $(\sim p \vee q) \to (r \to s)$

(c) $u \wedge \sim w$

(d) $t \to s$

(e) $(\sim t \wedge \sim r) \to w$

(f) $\therefore s$

For next time:

Pg 119: 3.8.(3 & 5)
Pg 122: 3.9.(3-7)

Read carefully 3.(10 & 11)
Skim 3.(12 & 13)

Take quiz