"Stand-alone topics" outline:

- Foldl, and how to model mathematical functions (last week Friday)
- Fixed-point iteration (**Today**)
- The Huffman encoding (Wednesday)
- Review for the final exam (Friday)

Today:

- Currying functions
- Specific problem: calculating square roots
- Medium-general problem: finding roots of real-valued functions
- Very general solution: Fixed-point iteration
- Applying general solution to specific problems

Final version of modeling project due Fri, Dec 8

arity, n The number of elements by virtue of which something is unary, binary, etc.

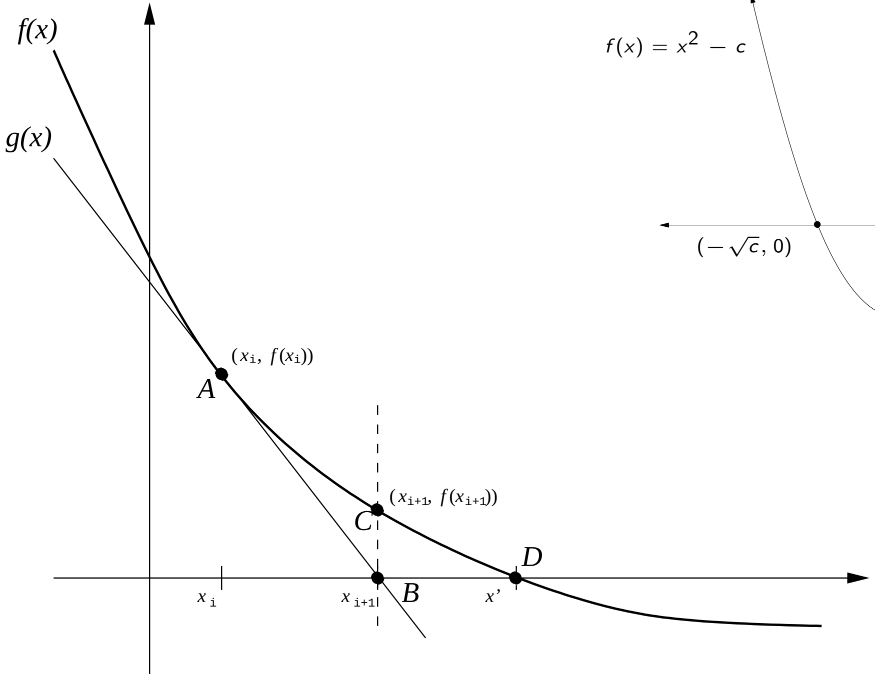
curry, v.1 (a) To rub down or dress (a horse etc.) with a comb. (b) (Applied to persons) to seek to win favor, or ingratiate oneself with another.

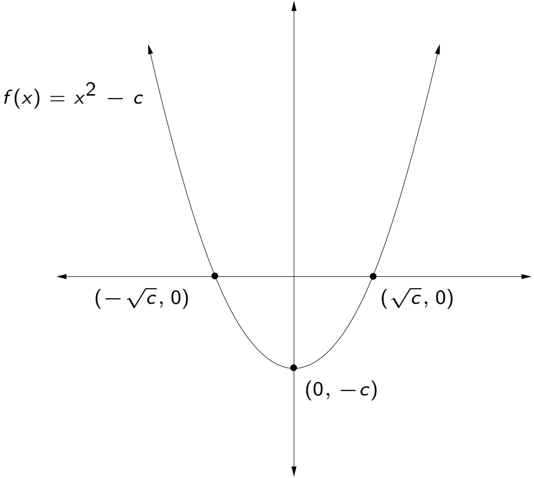
curry, v.2 *Obsolete.* To ride or run with haste or rapidity

curry, v.3 To flavor or prepare with curry or curry-powder.

curry, v.4 To transform a function that takes multiple arguments into a function that takes a single argument and returns a new function that takes the remainder of the arguments and returns the result.

Definition of *arity* and the first three definitions of *curry* are adapted from the Oxford English Dictionary. The fourth definition of *curry* is adapted from wiktionary.org.


f(x)
g(x)
(x_i, f(x_i))
A
(x_{i+1}, f(x_{i+1}))
C
D
B
x_i
x_{i+1}
x'


f(x) = x^2 − c
(−√c, 0)
(√c, 0)
(0, −c)

Point-slope form: Given a point (x_1, y_1) and a slope m,

$$\begin{aligned} \frac{y-y_1}{x-x_1} &= m \\ y - y_1 &= m \cdot (x - x_1) \\ y &= m \cdot (x - x_1) + y_1 \end{aligned}$$

To find **root** of function f near a guess x_i, draw a tangent line at $(x_1, f(x_1))$:

$$g(x) = f'(x_1) \cdot (x - x_1) + f(x_1)$$

Then find where this line hits the x-axis:

$$\begin{aligned} 0 &= f'(x_1)(x - x_1) + f(x_1) \\ x &= \frac{x_1 f'(x_1) - f(x_1)}{f'(x_1)} = x_1 - \frac{f(x_1)}{f'(x_1)} \end{aligned}$$

Consider that our *next guess*. In general,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

In general, to find a *new, improved guess* x_{i+1} for a root of function f from current guess x_i,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

To find $\sqrt{c}$, apply this to $f(x) = x^2 - c$.

$$f'(x) = 2x$$

And so the *guess-improving function* is

$$I(x) = x - \frac{x^2 - c}{2x}$$

Algorithm for finding an approximation for a root:

If our guess is good enough, then stop.
Otherwise, improve it and try again.

Fixed-point iteration. Algorithm for any iterative guess-improving process:

If our guess is good enough, then stop.
Otherwise, improve it and try again.

Elements:

- Guess-improver
- Current guess
- Good-enough tester

For finding roots using Newton's method, the guess-improver is

$$I(x) = x - \frac{f(x)}{f'(x)}$$

What if x is an exact root, that is, if $f(x) = 0$?

Lessons:

- Interaction between mathematics and computer science: An algorithm solves a mathematical problem.
- Interaction between continuous and discrete mathematics: The *continuous* functions are themselves *discrete* objects.
- Generalization in math and computer science: Functions and algorithms can be parameterized to become more general.

For next time:

Pg 375: 7.13.(1,3)

Skim 6.12, new version found on Canvas