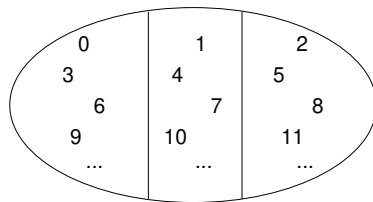
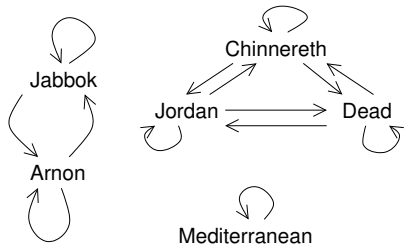


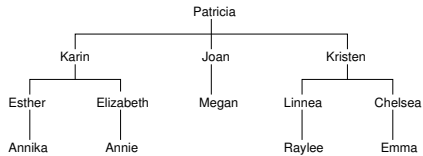
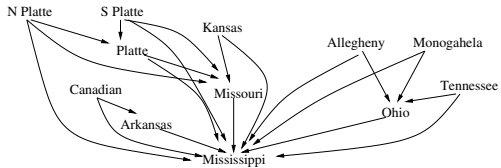
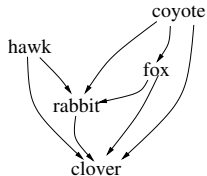
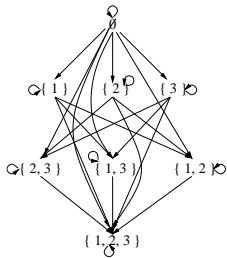
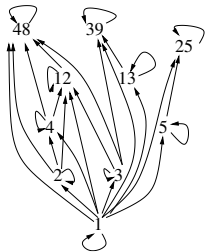
## Chapter 5 roadmap:

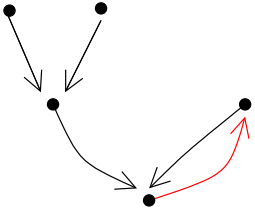
- ▶ Introduction to relations (Monday, Oct 9)
- ▶ Properties of relations (Wednesday, Oct 11, and Friday, Oct 13)
- ▶ Transitive closure (last week Friday)
- ▶ Partial order relations (**Today**)
- ▶ Review for Test 2 (Wednesday)
- ▶ Test 2 on Chapters 4 & 5 (Friday)

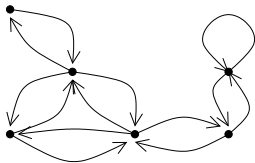
## Today:

- ▶ Antisymmetry
- ▶ Partial order relations
- ▶ Topological sort



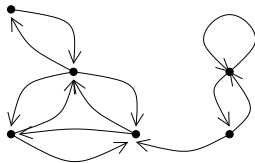






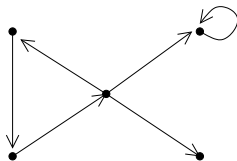
symmetric

All arrows  
have a back arrow.



asymmetric  
(not symmetric)

There exists an arrow  
without a back arrow.



antisymmetric

("very" not symmetric)  
No arrows have back arrows  
except self loops.

Formal definition:

*A relation  $R$  on a set  $X$  is antisymmetric if  $\forall x, y \in X$ , if  $(x, y) \in R$  and  $(y, x) \in R$ , then  $x = y$ .*

Informal definition:

*If both an arrow and its reverse exist in an antisymmetric relation  $R$ , then that arrow must be a self loop (and, hence, it is its own reverse).*

Alternate formal definition:

*A relation  $R$  on a set  $X$  is antisymmetric if  $\forall (x, y) \in R$ , either  $x = y$  or  $(y, x) \notin R$ .*

Rock beats scissors; scissors beats paper; paper beats rock.

Grasshopper eats corn; mouse eats corn; mouse eats grasshopper; snake eats mouse; hawk eats mouse; hawk eats snake.

Aurelia is better than Gwendolyn at pitching; Gwendolyn is better than Aurelia at batting.

Peter Pan is shorter than Treasure Island; Treasure Island is shorter than Anna Karenina; Anna Karenina is shorter than The Count of Monte Christo.

CSCI 235 is a prereq for CSCI 245; CSCI 245 is a rereq for CSCI 345; CSCI 243 is a prereq for CSCI 345; MATH 231 is a prereq for MATH 245; CSCI 345 is a prereq for CSCI 381; MATH 245 is a prereq for CSCI 381.

I married a widow with a grown daughter; my father, a widower, then married my step-daughter. Thus I am my own step-grampa. (The relation in this example is "is biological ancestor of or step-ancestor of".)

A relation  $R$  on a set  $X$  is antisymmetric if  $\forall x, y \in X$ , if  $(x, y) \in R$  and  $(y, x) \in R$ , then  $x = y$ .

**Ex 5.8.9.** Prove that  $|$  (divides) on  $\mathbb{N}$  is antisymmetric.

**Proof.** Suppose  $x, y \in \mathbb{N}$ ,  $x|y$ , and  $y|x$  (that is,  $(x, y), (y, x) \in |$ ). By definition of divides, there exists  $i, j \in \mathbb{N}$  such that

$$\begin{aligned}x &= i \cdot y \\ y &= j \cdot x\end{aligned}$$

Then

$$\begin{aligned}x &= i \cdot j \cdot x && \text{by substitution} \\ 1 &= i \cdot j && \text{by cancellation} \\ i &= j = 1 && \text{by arithmetic} \\ x &= y && \text{by identity}\end{aligned}$$

Therefore  $|$  is antisymmetric by definition.  $\square$



Antisymmetry:

A relation  $R$  on a set  $X$  is *antisymmetric* if  $\forall x, y \in X$ , if  $(x, y) \in R$  and  $(y, x) \in R$ , then  $x = y$ .

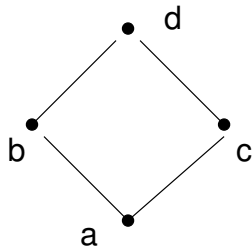
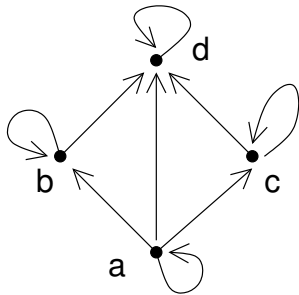
Partial order relation:

A *partial order relation* (or just *partial order*) is a relation that is reflexive, transitive, and antisymmetric.

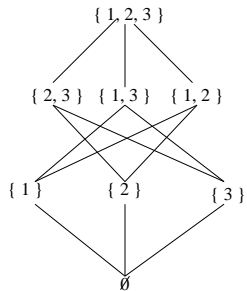
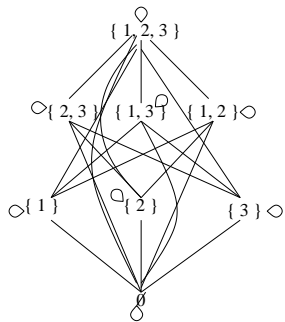
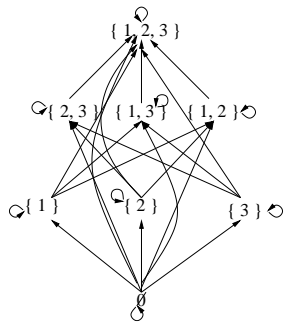
A *strict partial order (relation)* is a relation that is irreflexive, transitive and antisymmetric.

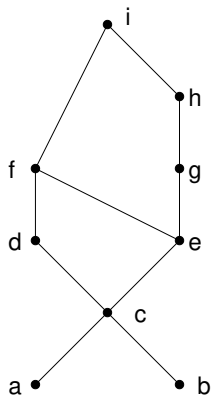
Partially ordered set:

A *partially ordered set* or *poset* is a set together with a partial order on that set.



$$R = \{(a, a), (a, b), (a, c), (a, d), (b, b), (b, d), (c, c), (c, d), (d, d)\}$$





Comparable:  $a \preceq c, d \preceq f, e \preceq f, e \preceq h, c \preceq i$

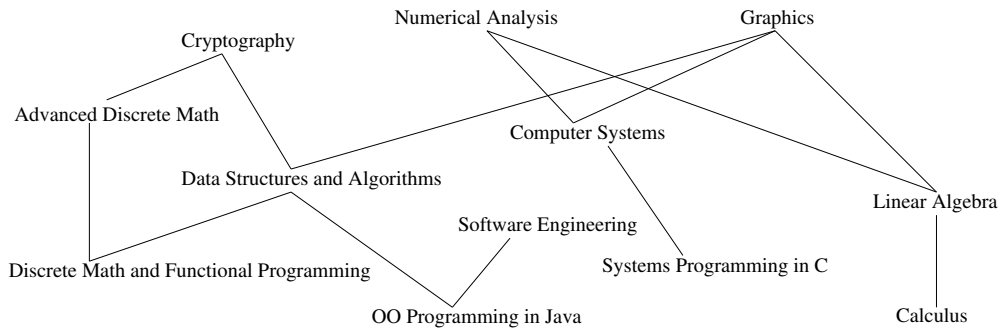
Not comparable:  $a$  and  $b$ ;  $d$  and  $e$ ;  $f$  and  $h$

Maximal and greatest:  $i$

Minimal:  $a$  and  $b$

No least

Everyday examples: Preparing a meal, writing a term paper, getting dressed



A partial order  $R$  on a set  $X$  is a *total order* if for all  $x, y \in X$ , either  $x \preceq y$  or  $y \preceq x$ , that is,  $x$  and  $y$  are comparable.

Standard example of a total order:  $\leq$ .

A *partial order relation* (or just *partial order*) is a relation that is reflexive, transitive, and antisymmetric.

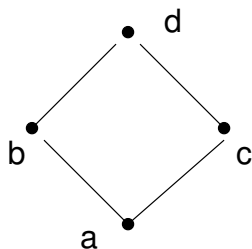
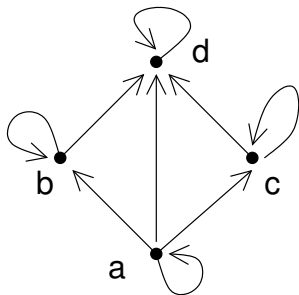
A partial order  $R$  on a set  $X$  is a *total order* if for all  $x, y \in X$ , either  $x \preceq y$  or  $y \preceq x$ , that is,  $x$  and  $y$  are comparable.

A *topological sort* of a partial order  $R$  is a total order that is a superset of  $R$ .

| (divides)             $\leq$

is prerequisite for    Ralph takes before

can put on before    you put on before



$$R = \{(a, a), (a, b), (a, c), (a, d), (b, b), (b, d), (c, c), (c, d), (d, d)\}$$

A topological sort for  $R$ :  $R \cup \{(b, c)\}$ , written as  $a, b, c, d$

Another topological sort for  $R$ :  $R \cup \{(c, b)\}$ , written as  $a, c, b, d$



**For next time:**

*Pg 226: 5.8.(1-5)*

*Pg 231 5.9.(1 & 8)*

**For Monday, Oct 30:**

*Read 6.(1-3)*

*Take quiz*