

Semester roadmap:

Ch 1 & 2: Raw materials

Ch 3: Formal logic

—Test 1, Sept 25 —

Ch 4: Proofs

Ch 5: Relations

— Test 2, Oct 27 —

Ch 6: Self reference

Ch 7: Functions

— Test 3, Nov 29 —

Chapter 6 roadmap:

- ▶ Recursive definitions, recursive types (**Today**)
- ▶ Recursive proofs I: Structural induction (Wednesday)
- ▶ Recursive proofs II: Mathematical induction (Friday)
- ▶ Recursive proofs III: Loop invariants (next week Monday and Wednesday)

Project prototype due Wed, Nov 8

Axiom 7

There exists a whole number 0.

Axiom 8

Every whole number n has a successor, $\text{succ } n$.

Axiom 9

No whole number has 0 as its successor.

Axiom 10

If $a, b \in \mathbb{W}$, then $a = b$ iff $\text{succ } a = \text{succ } b$.

A whole number is either zero or one more than another whole number.

Compare to:

A list is either empty or an element together with its following list.

5 is a whole number because

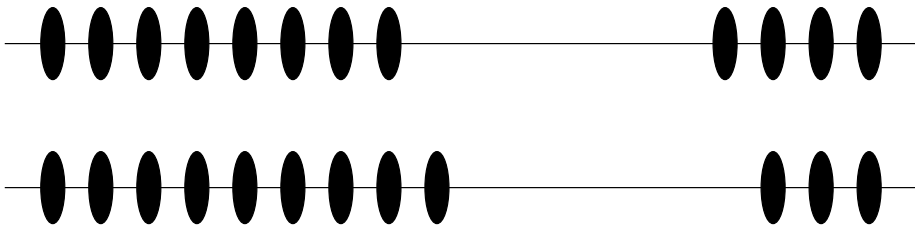
5 is a whole number because it is the successor of 4, which is a whole number because

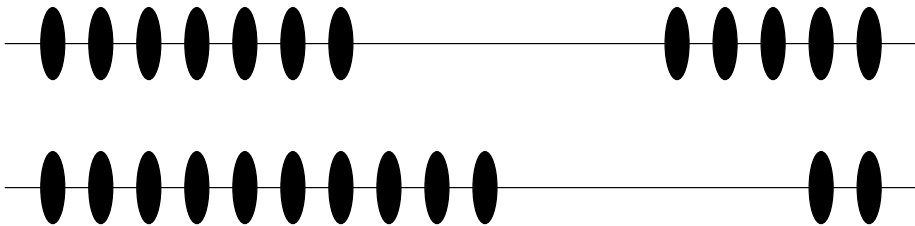
5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because

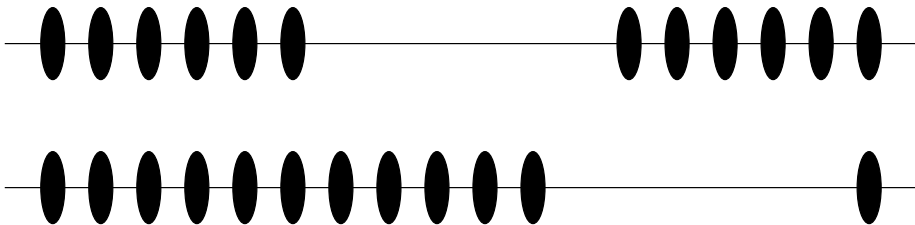
5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because it is the successor of
2, which is a whole number because

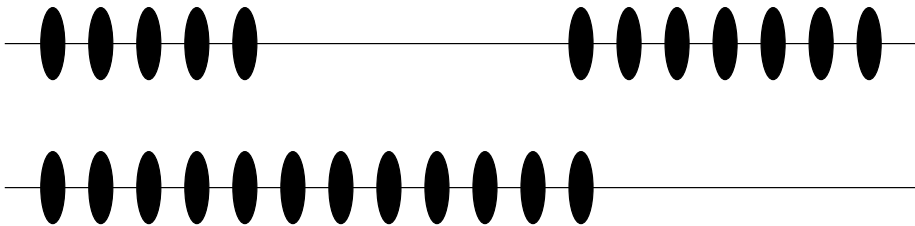
5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because it is the successor of
2, which is a whole number because it is the successor of
1, which is a whole number because

5 is a whole number because it is the successor of
4, which is a whole number because it is the successor of
3, which is a whole number because it is the successor of
2, which is a whole number because it is the successor of
1, which is a whole number because it is the successor of
0, which is a whole number by Axiom 7.









Lemmas for addition:

- ▶ $0 + b = b$
- ▶ $a + 0 = a$
- ▶ $a + b = (a + 1) + (b - 1)$

Lemmas for subtraction:

- ▶ $a - 0 = a$
- ▶ $a - b = (a - 1) - (b - 1)$

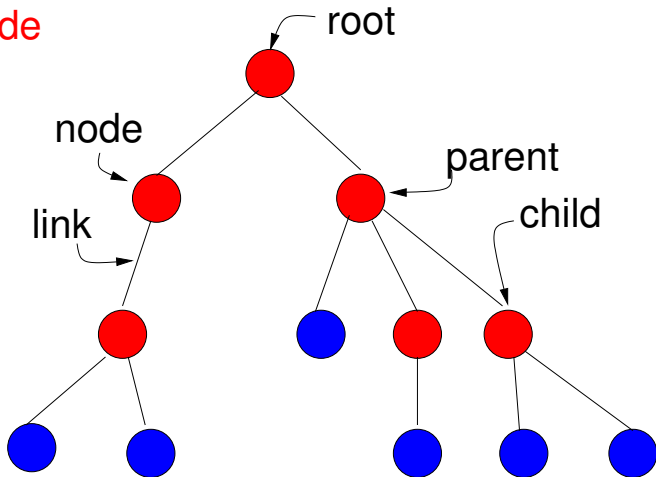
Lemmas for multiplication:

- ▶ $a \cdot 0 = 0$
- ▶ $0 \cdot b = 0$
- ▶ $a \cdot 1 = a$
- ▶ $a \cdot b = a + (a \cdot (b - 1))$

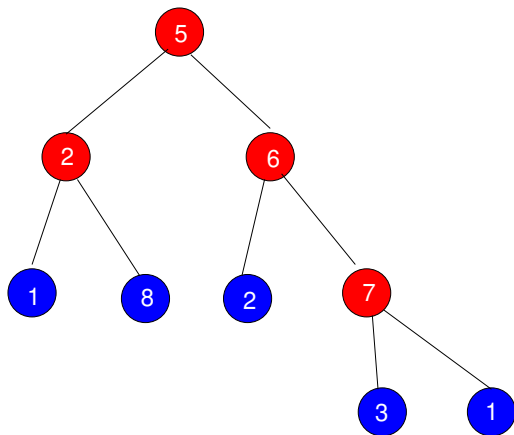
Tree

internal node

leaf



Full Binary Tree

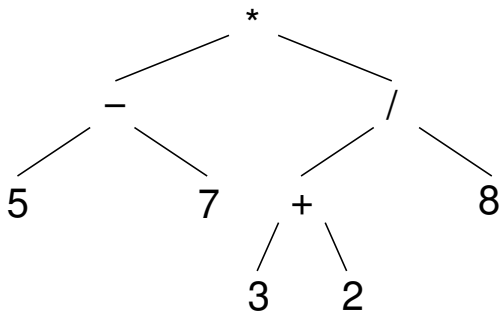


Expression trees:

```
datatype operation = Plus | Minus | Mul | Div;  
datatype expression = Internal of operation * expression * expression  
                    | Leaf of int;
```

$((5 - 7) * ((3 + 2)/8))$

```
val exprExample = Internal(Mul, Internal(Minus, Leaf(5), Leaf(7)),  
                           Internal(Div,  
                                   Internal(Plus, Leaf(3),  
                                           Leaf(2)),  
                                   Leaf(8)));
```



For next time:

Pg 260: 6.2.(6-8, 14-17)

Read 6.4

Take quiz