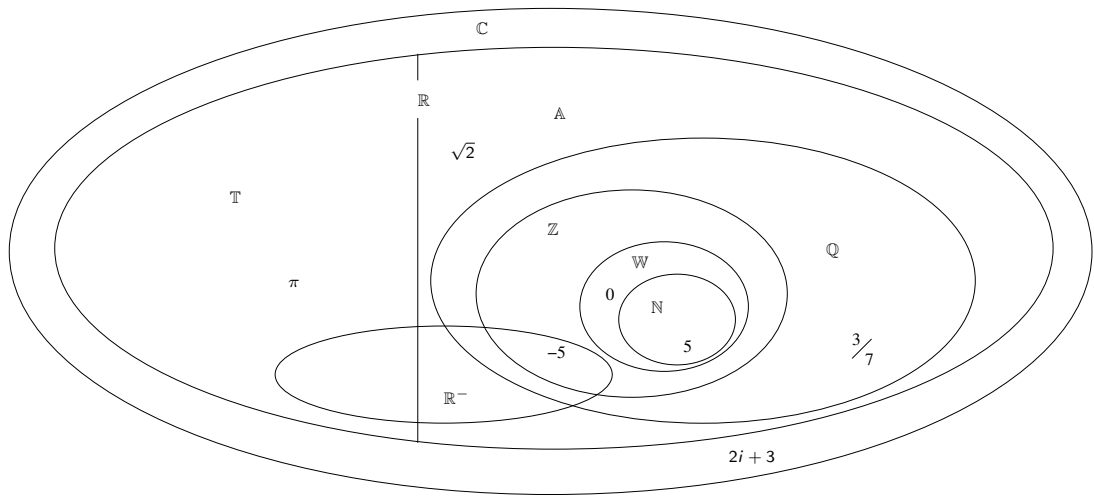


Chapter 1 outline:

- ▶ Introduction, sets and elements (this past Wednesday)
- ▶ Set operations; visual verification of set propositions (**Today**)
- ▶ Introduction to SML; cardinality and Cartesian products (next week Monday)
- ▶ Making types in SML (next week Wednesday)
- ▶ Making functions in SML (next week Friday)

Today:

- ▶ Set symbols and terminology
- ▶ Set notation
- ▶ Set operations
- ▶ Verifying set equivalence visually



5 is a natural number; *or* the collection of natural numbers contains 5. $5 \in \mathbb{N}$

Adding 0 to the collection of natural numbers makes the collection of whole numbers. $\mathbb{N} \cup \{0\} = \mathbb{W}$

Merging the algebraic numbers and the transcendental numbers makes the real numbers. $\mathbb{A} \cup \mathbb{T} = \mathbb{R}$

Transcendental numbers are those real numbers which are not algebraic numbers. $\mathbb{T} = \mathbb{R} - \mathbb{A}$

Nothing is both transcendental and algebraic, *or* the collection of things both transcendental and algebraic is empty. $\mathbb{T} \cap \mathbb{A} = \emptyset$

Negative integers are both negative and integers. $\mathbb{Z}^- = \mathbb{Z} \cap \mathbb{R}^-$

All integers are rational numbers. $\mathbb{Z} \in \mathbb{R}$

Since all rational numbers are algebraic numbers and all algebraic numbers are real numbers, it follows that all rational numbers are real numbers.

$$\begin{aligned} \mathbb{Q} &\subseteq \mathbb{A} \\ \mathbb{A} &\subseteq \mathbb{R} \\ \therefore \mathbb{Q} &\subseteq \mathbb{R} \end{aligned}$$

Axiom (Existence.)

There is a set with no elements.

Axiom (Extensionality.)

*If every element of a set X is an element of a set Y
and every element of Y is an element of X , then $X = Y$.*

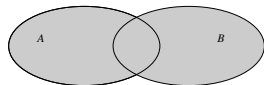
Union

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

$$\{1, 2, 3\} \cup \{2, 3, 4\} = \{1, 2, 3, 4\}$$

$$\{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\}$$

$$\{1, 2\} \cup \{1, 2, 3\} = \{1, 2, 3\}$$



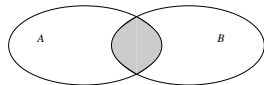
Intersection

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

$$\{1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}$$

$$\{1, 2\} \cap \{3, 4\} = \emptyset$$

$$\{1, 2\} \cap \{1, 2, 3\} = \{1, 2\}$$



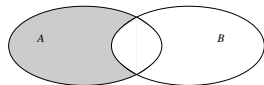
Difference

$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$$

$$\{1, 2, 3\} - \{2, 3, 4\} = \{1\}$$

$$\{1, 2\} - \{3, 4\} = \{1, 2\}$$

$$\{1, 2\} - \{1, 2, 3\} = \emptyset$$



1. $\{1, 2, 3, 4, 5\} \cup \{5, 6, 7\} =$

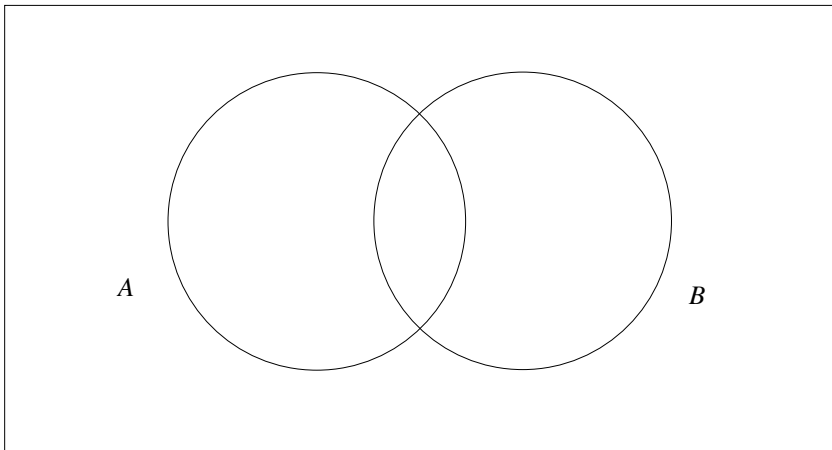
2. $\{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\} =$

3. $\{1, 2, 3, 4, 5\} - \{2, 3, 4\} =$

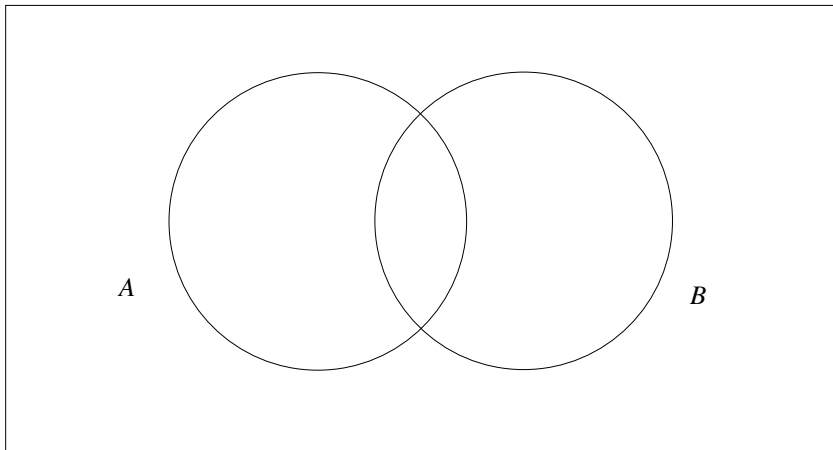
4. $\{1, 2, 3, 4, 5\} - \{3, 4, 5, 6, 7\} =$

Which of the following are equal to $\{1, 2, 3, 4\}$?

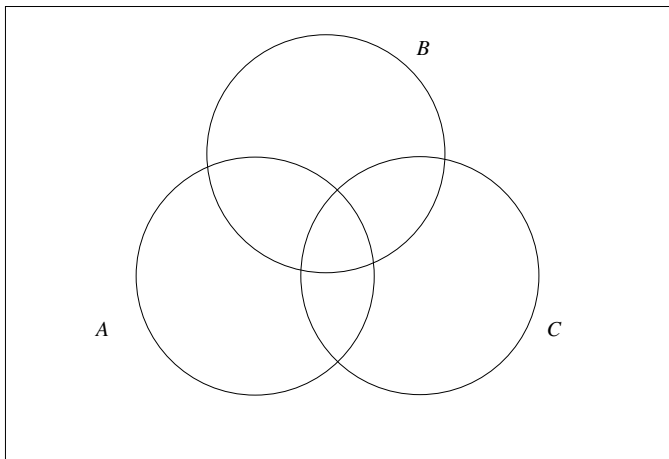
- ▶ $\{1, 2\} \cup \{3, 4\}$
- ▶ $\{1, 2, 3\} \cup \{4\}$
- ▶ $\{1, 2, 3\} \cup \{2, 3, 4\}$
- ▶ $\{1, 2, 3\} \cup \{3, 4, 5\}$
- ▶ $\{2, 3\} \cup \{1, 4\}$
- ▶ $\{1\} \cup \{3, 4\}$
- ▶ $\{4, 3, 2, 1\}$
- ▶ $\{1\} \cup \{1, 2\} \cup \{1, 2, 3\} \cup \{1, 2, 3, 4\}$



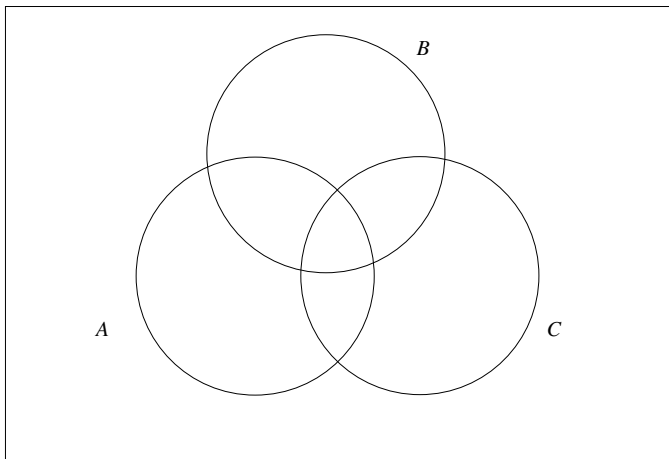
1.4.7. $(A \cap B) - A$



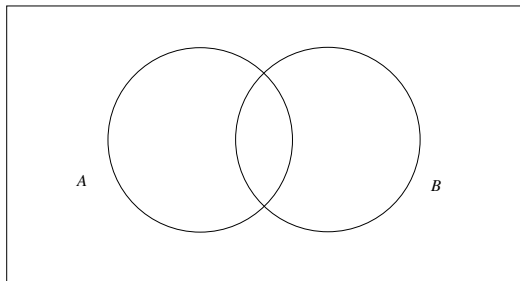
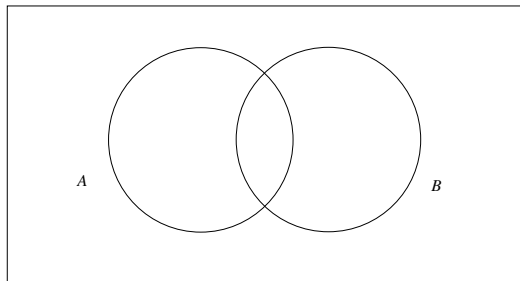
1.4.8. $(A - B) \cup (B - A)$



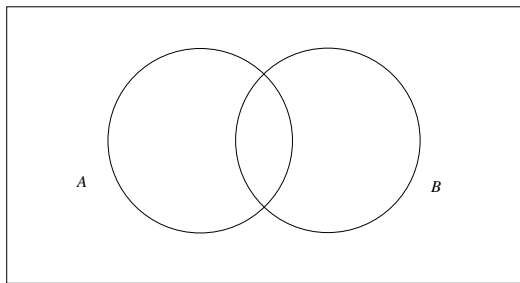
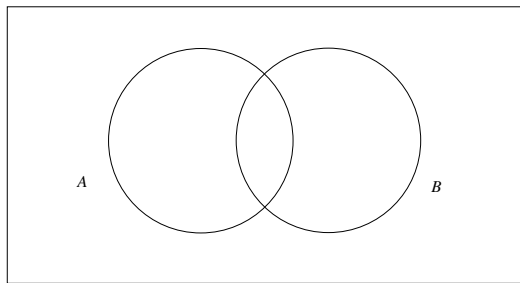
1.4.9. $(A \cup B) \cap (A \cup C)$



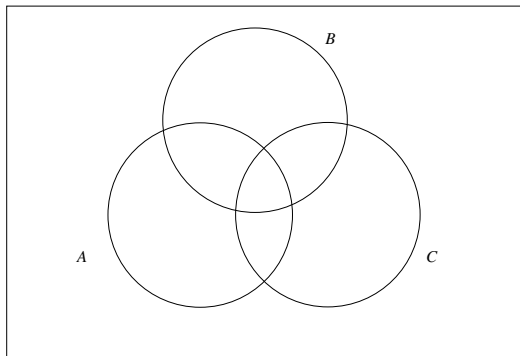
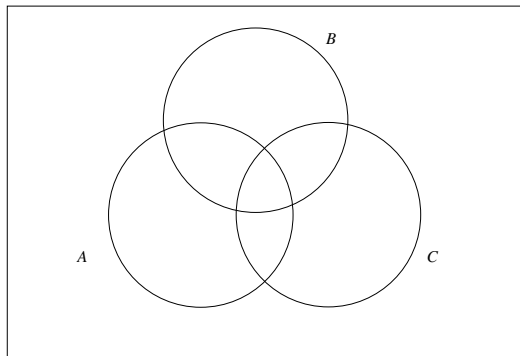
1.4.10. $\overline{(A \cap B)} \cap (A \cup C)$



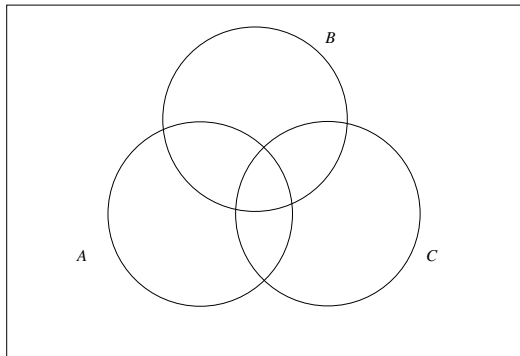
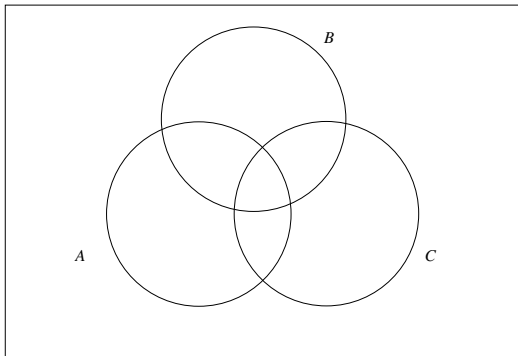
$$A \cup (A \cap B) = A$$



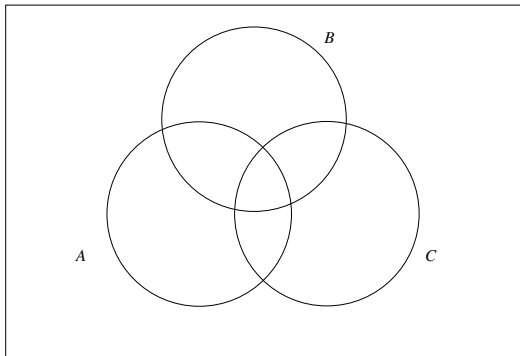
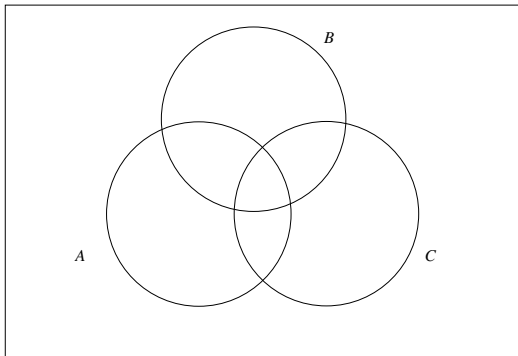
$$A \cup \bar{A} = \mathcal{U}$$



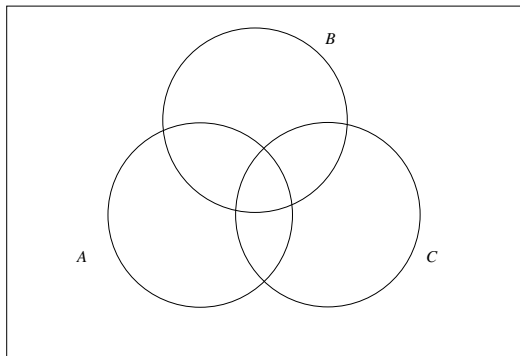
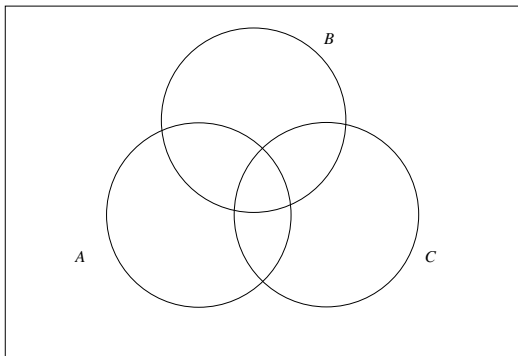
$$A \cup (B \cup C) = (A \cup B) \cup C$$



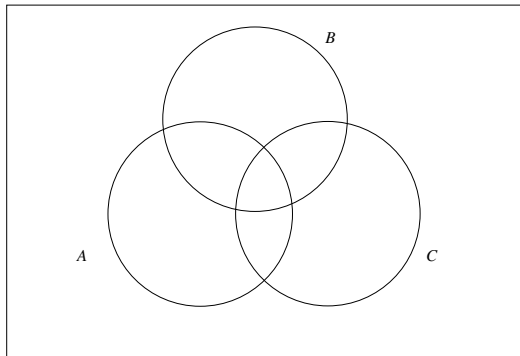
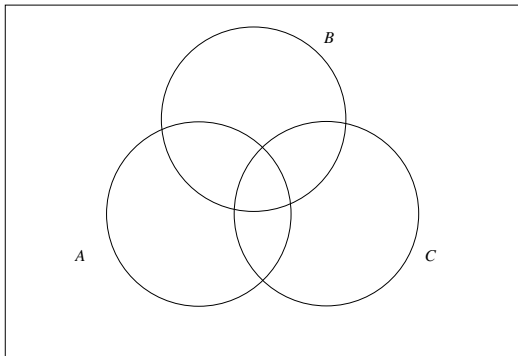
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



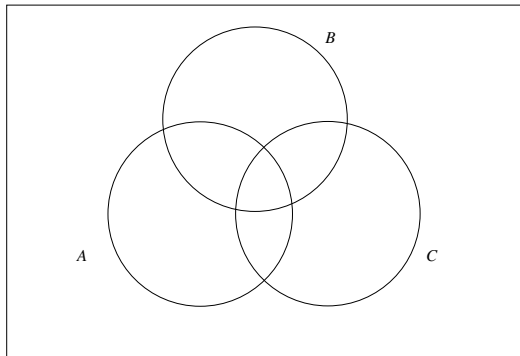
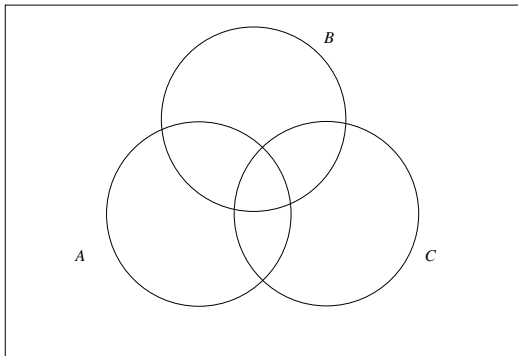
$$A \cap B = A - (A - B)$$



$$(A \cap C) - (C - B) = A \cap B \cap C$$



$$A \cup (A - B) = A$$



$$(A \cup (B - C)) \cap \overline{B} = A - B$$

For next time:

Pg 12: 1.3.(11-14, 16)

Pg 16: 1.4.(1-6, 19)

Pg 20: 1.5.(8-11)

Read 1.(6-9)

Take quiz