

Chapter 6 outline:

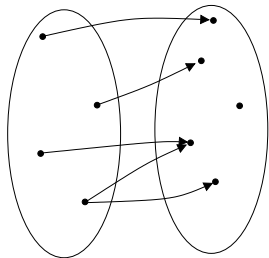
- ▶ Introduction, function equality, and anonymous functions (last week Wednesday)
- ▶ Image and inverse images (last week Friday)
- ▶ Function properties and composition (Monday)
- ▶ Map, reduce, filter (Wednesday)
- ▶ Cardinality (**Today**)
- ▶ Countability **plus practice quiz** (next week Monday, Nov 18)
- ▶ Review (next week Wednesday, Nov 20)
- ▶ Test 3, on Ch 5 & 6 (next week Friday, Nov 22)

Today:

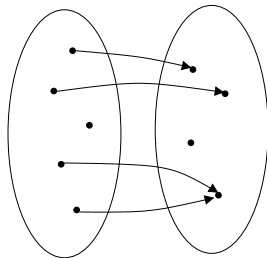
- ▶ Hints on previous HW problems
- ▶ Programming topic from last time
- ▶ Formal definition of cardinality
- ▶ If $A \cap B = \emptyset$, then $|A \cup B| = |A| + |B|$
- ▶ If $f : A \rightarrow B$ is one-to-one, then $|A| \leq |B|$.
- ▶ If $|A| > |B|$, and $f : A \rightarrow B$, then f is not one-to-one.

Ex. 6.3.3. If $A, B \subseteq X$ and f is one-to-one, then $F(A - B) \subseteq F(A) - F(B)$.

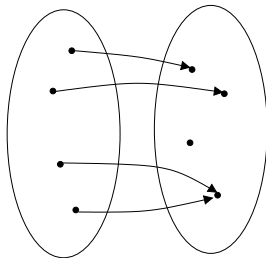
Ex. 6.4.1. If $f : X \rightarrow Y$, then $f \circ i_X = f$.



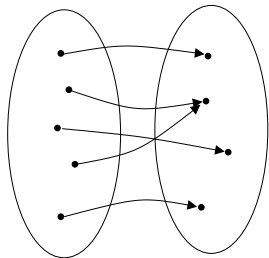
Not a function



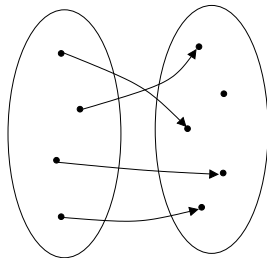
Not a function



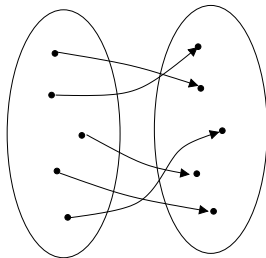
A function but not
one-to-one or onto



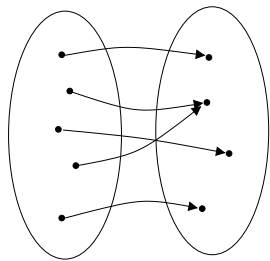
Onto, not one-to-one



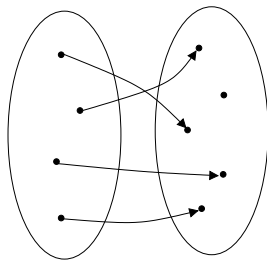
One-to-one, not onto



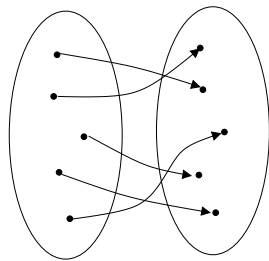
One-to-one correspondence



Onto, not one-to-one
 $|X| \geq |Y|$



One-to-one, not onto
 $|X| \leq |Y|$



One-to-one correspondence
 $|X| = |Y|$

Two finite sets X and Y have the *the same cardinality* as each other if there exists a one-to-one correspondence from X to Y .

To use this *analytically*:

Suppose X and Y have the same cardinality. Then let f be a one-to-one correspondence from X to Y .

f is both onto and one-to-one.

To use this *synthetically*:

Given sets X and $Y \dots$

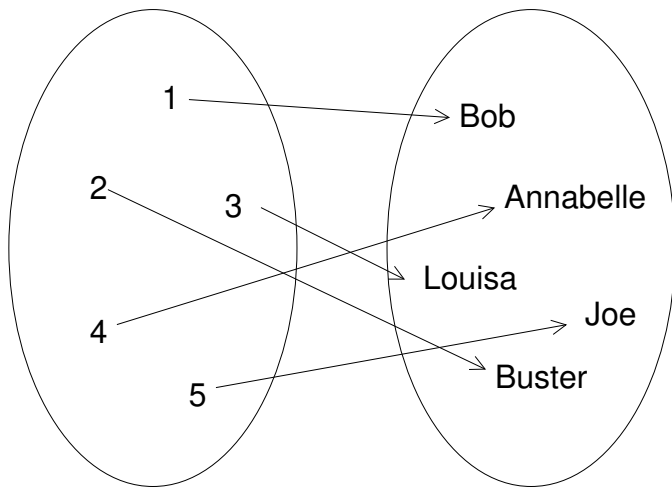
[Define f] Let $f : X \rightarrow Y$ be a function defined as \dots

Suppose $y \in Y$. *Somehow find* $x \in X$ such that $f(x) = y$. Hence f is onto.

Suppose $x_1, x_2 \in X$ such that $f(x_1) = f(x_2)$. *Somehow show* $x_1 = x_2$. Hence f is one-to-one.

Since f is a one-to-one correspondence, X and Y have the same cardinality as each other.

A finite set X has cardinality $n \in \mathbb{N}$, which we write as $|X| = n$, if there exists a one-to-one correspondence from $\{1, 2, \dots, n\}$ to X . Moreover, $|\emptyset| = 0$.



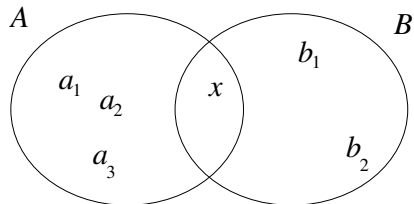
Theorem 6.12. If A and B are finite, disjoint sets, then $|A \cup B| = |A| + |B|$.

Theorem 6.13. If A and B are finite sets and $f : A \rightarrow B$ is one-to-one, then $|A| \leq |B|$.

Theorem 6.14. If A and B are finite sets, $|A| > |B|$, and $f : A \rightarrow B$, then f is not one-to-one.

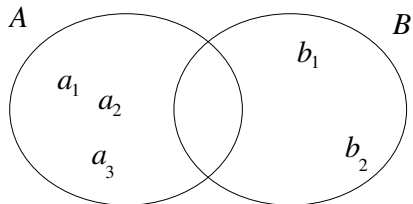
Exercise 6.6.5. If A and B are finite sets and $f : A \rightarrow B$ is onto, then $|A| \geq |B|$.
(Unassigned)

$$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$$



$$|A \cup B| = |\{a_1, a_2, a_3, x, b_1, b_2\}| = 6$$

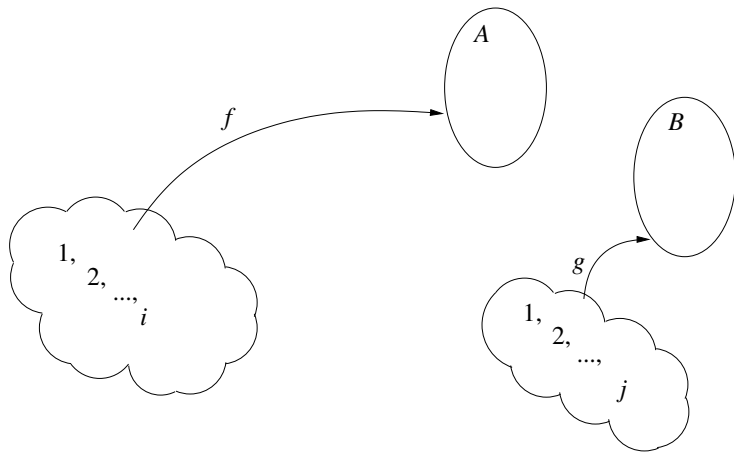
$$\begin{aligned} |A| + |B| &= \\ &= |\{a_1, a_2, a_3, x\}| + |\{x, b_1, b_2\}| \\ &= 4 + 3 = 7 \end{aligned}$$



$$|A \cup B| = |\{a_1, a_2, a_3, b_1, b_2\}| = 5$$

$$\begin{aligned} |A| + |B| &= \\ &= |\{a_1, a_2, a_3\}| + |\{b_1, b_2\}| \\ &= 3 + 2 = 5 \end{aligned}$$

$$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$$



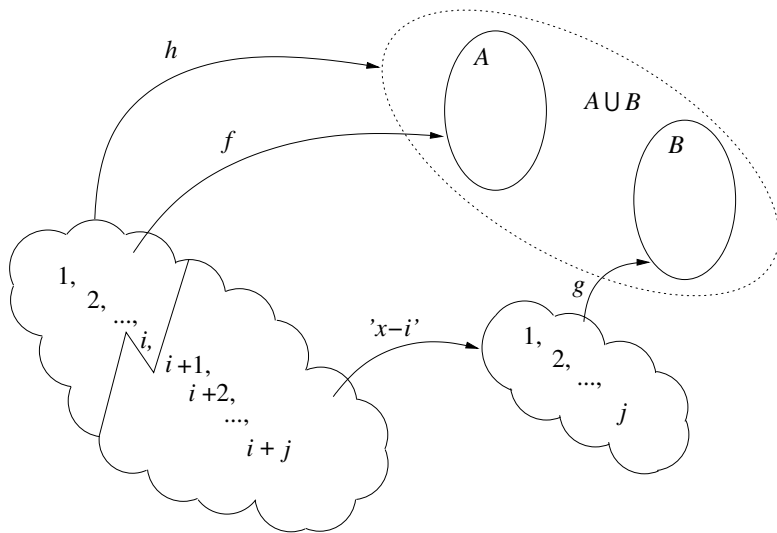
$$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$$

x	f
1	Zed
2	Yelemis
3	Xavier

x	g
1	Wilhelmina
2	Valerie
3	Ursula
4	Tassie

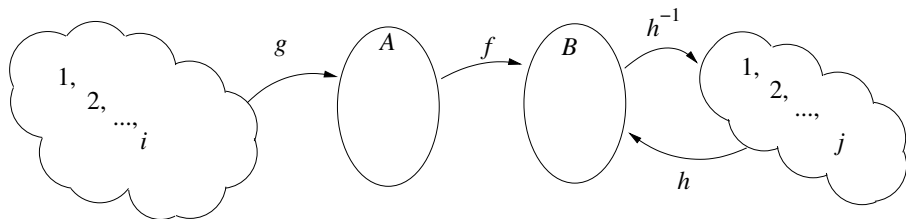
x	h
1	$f(1) = \text{Zed}$
2	$f(2) = \text{Yelemis}$
3	$f(3) = \text{Xavier}$
4	$g(4-3) = g(1) = \text{Wilhelmina}$
5	$g(5-3) = g(2) = \text{Valerie}$
6	$g(6-3) = g(3) = \text{Ursula}$
7	$g(7-3) = g(4) = \text{Tassie}$

$$A \cap B = \emptyset \rightarrow |A \cup B| = |A| + |B|$$

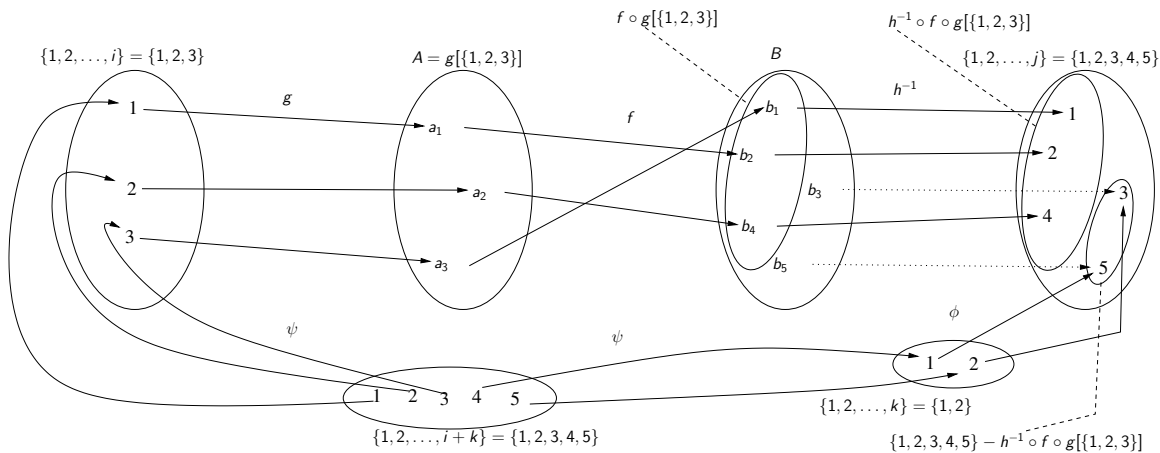


$$f : A \rightarrow B \text{ is one-to-one} \rightarrow |A| \leq |B|$$

Theorem 6.13. If A and B are finite sets and $f : A \rightarrow B$ is one-to-one, then $|A| \leq |B|$.



$f : A \rightarrow B$ is one-to-one $\rightarrow |A| \leq |B|$



Theorem 6.13. If A and B are finite sets and $f : A \rightarrow B$ is one-to-one, then $|A| \leq |B|$.

Theorem 6.14. If A and B are finite sets, $|A| > |B|$, and $f : A \rightarrow B$, then f is not one-to-one.



Art credit: Sharon Dunbar '23

For next time:

Do Exercises 6.6.(1 & 2). Both of these are simpler than they appear. See Canvas for hints/clarifications.

No reading or quiz.