

## Chapter 4 roadmap:

- ▶ Subset proofs (Monday)
- ▶ Set equality and emptiness proofs (**Today**)
- ▶ Conditional and biconditional proofs (Friday)
- ▶ Proofs about powersets (new week Monday)
- ▶ (Test 2, Friday, Oct 18)

## Today:

- ▶ Proofs that sets are equal
- ▶ Proofs that a set is empty

## General forms:

### 1. Facts ( $p$ )

#### Set forms

1. Subset  $X \subseteq Y$
2. Set equality  $X = Y$
3. Set emptiness  $X = \emptyset$

### 2. Conditionals ( $p \rightarrow q$ )

### 3. Biconditionals ( $p \leftrightarrow q$ )

$$A \times (B - C) \subseteq (A \times B) - (A \times C).$$

**Proof (long version).** Suppose  $x \in A \times (B - C)$ . By definition of Cartesian product,  $x = (a, d)$  for some  $a \in A$  and  $d \in B - C$ . By definition of difference,  $d \in B$  and  $d \notin C$ .

By definition of Cartesian product,  $(a, d) \in A \times B$ . Also by definition of Cartesian product, this time used negatively,  $(a, d) \notin A \times C$ .

[That is, we rewrite  $d \notin C$  as  $\sim (d \in C)$ . By generalization,  $\sim (d \in C \wedge a \in A)$ . By definition of Cartesian product,  $\sim ((a, d) \in A \times C)$ . This can be rewritten as  $(a, d) \notin A \times C$ .]

By definition of difference,  $(a, d) \in (A \times B) - (A \times C)$ . By substitution,  $x \in (A \times B) - (A \times C)$ . Therefore, by definition of subset,  $A \times (B - C) \subseteq (A \times B) - (A \times C)$ .  $\square$

$$A \times (B - C) \subseteq (A \times B) - (A \times C).$$

**Proof (short version).** Suppose  $(a, d) \in A \times (B - C)$ . By definition of Cartesian product,  $a \in A$  and  $d \in B - C$ .

By definition of difference,  $d \in B$  and  $d \notin C$ . By definition of Cartesian product,  $(a, d) \in A \times B$  and  $(a, d) \notin A \times C$ .

By definition of difference,  $(a, d) \in (A \times B) - (A \times C)$ . Therefore, by definition of subset,  $A \times (B - C) \subseteq (A \times B) - (A \times C)$ .  $\square$

**For next time:**

*Pg 160: 4.2.(3, 14, 15, 18)*

*Pg 161: 4.3.(5 & 6)*

*See assignment on Canvas for hint on Ex 4.2.15.*

*Read 4.(4 & 5)*

*Take quiz*