Chapter 3 roadmap:

Propositions, booleans, logical equivalence. §3.(1 & 2) (last week Friday)

- Conditional propositions, conditional expressions. §3.3 (Monday)
- Arguments and predicates. §3.(5 & 6) (Wednesday)
- Predicates and quantification. §3.(6 & 7) (Today)
- (Begin proofs next week)

Today:

- Predicates
  - Definition (review)
  - Predicates in set-builder notation
  - Predicates in code
- Quantification
  - Universal quantification
  - Existential quantification
  - Quantification in code
  - Multiple quantification

Four ways to interpret/define the idea of a *predicate* 

A predicate is a proposition with a parameter.

x < 5 x is orange

A predicate is a function whose value is true or false.

$$P(x) = x < 5$$
  $Q(x) = x$  is orange

A predicate is a part of a sentence that complements a noun phrase to make a proposition.

A pumpkin is orange.

► A predicate is a truth set  $P: \mathbb{N} \to \mathbb{B}, P(x) = x < 5$ Truth set: {1,2,3,4} Q(x) = x is orange Q(x) = x is orange Q(x) = x is orange Universal quantification

"For all multiples of 3, the sum of their digits is a multiple of 3."

Let *D* be the set of multiples of 3, that is  $D = \{n \in \mathbb{N} \mid n \mod 3 = 0\} = \{3, 6, 9, 12, 15, 18, \ldots\}$ 

 $\forall \ x \in D, \mathtt{sum}(\mathtt{digify}(x)) \in D$ 

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Other examples:

- ▶  $\forall x \in \{5, 7, 19, 23, 43\}$ , x is prime.
- ▶  $\forall x \in \{4, 16, 25, 31\}$ , x is a perfect square.

Existential quantification

"There is a multiple of 3 that is not a perfect square."

 $\exists x \in D \mid x \text{ is not a perfect square}$ 

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Alternately, "Some multiples of 3 are not perfect squares."

General forms for universal and existential quantification:

$$\forall x \in X, P(x) \qquad \exists x \in X \mid P(x)$$

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 $\forall x \in \emptyset, P(x)$  is always (vacuously) true.

 $\exists x \in \emptyset \mid P(x)$  is always false

$$\sim (\forall \ x \in X, P(x))$$
  
$$\equiv \ \sim (P(x_1) \land P(x_2) \land \cdots)$$
  
$$\equiv \ \sim P(x_1) \lor \sim P(x_2) \lor \cdots$$
 By DeMorgan's Law

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$$\equiv \exists x \in X \mid \sim P(x)$$

Т	S	R	Q	Р
K	L	М	N	О
J	I	Н	G	F
Е	D	С	В	А

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- 1. Bob passed through P.
- 2. Bob passed through N.
- 3. Bob passed through M.
- 4. If Bob passed through O, then Bob passed through F.
- 5. If Bob passed through K, then Bob passed through L.
- 6. If Bob passed through L, then Bob passed through K.

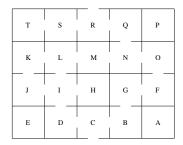
Let X be the routes through the maze, that is,  $X = \{CBGFONQR, CDILMNQR, CDIJKLMNQR\}$ 

Let P(x) = route x contains L, Q(x) = route x contains K.

Consider  $\forall x \in X, P(x) \rightarrow Q(x)$ .

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X	P(x)	Q(x)	$P(x) \rightarrow Q(x)$
CBGFONQR			
CDILMNQR			
CDIJKLMNQR			



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## For next time:

Do Exercises 3.6.(2, 5-9) and 3.7.(20-24) Read Section 4.1 Take quiz

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