

Chapter 3 roadmap:

- ▶ Propositions, booleans, logical equivalence. §3.(1 & 2) (last week Friday)
- ▶ Conditional propositions, conditional expressions. §3.3 (Monday)
- ▶ Arguments and predicates. §3.(5 & 6) (Wednesday)
- ▶ Predicates and quantification. §3.(6 & 7) (**Today**)
- ▶ (Begin proofs next week)

Today:

- ▶ Predicates
 - ▶ Definition (review)
 - ▶ Predicates in set-builder notation
 - ▶ Predicates in code
- ▶ Quantification
 - ▶ Universal quantification
 - ▶ Existential quantification
 - ▶ Quantification in code
 - ▶ Multiple quantification

Four ways to interpret/define the idea of a *predicate*

- ▶ A predicate is a proposition with a parameter.

$$x < 5 \quad x \text{ is orange}$$

- ▶ A predicate is a function whose value is true or false.

$$P(x) = x < 5 \quad Q(x) = x \text{ is orange}$$

- ▶ A predicate is a part of a sentence that complements a noun phrase to make a proposition.

A pumpkin **is orange**.

- ▶ A predicate is a truth set

$$\begin{array}{ll} P : \mathbb{N} \rightarrow \mathbb{B}, P(x) = x < 5 & Q(x) = x \text{ is orange} \\ \text{Truth set: } \{1, 2, 3, 4\} & \{ \text{pumpkin, fall leaves, orange juice, } \dots \} \end{array}$$

Universal quantification

“For all multiples of 3, the sum of their digits is a multiple of 3.”

Let D be the set of multiples of 3, that is

$$D = \{n \in \mathbb{N} \mid n \bmod 3 = 0\} = \{3, 6, 9, 12, 15, 18, \dots\}$$

$$\forall x \in D, \text{sum}(\text{digify}(x)) \in D$$

Other examples:

- ▶ $\forall x \in \{5, 7, 19, 23, 43\}$, x is prime.
- ▶ $\forall x \in \{4, 16, 25, 31\}$, x is a perfect square.

Existential quantification

“There is a multiple of 3 that is not a perfect square.”

$$\exists x \in D \mid x \text{ is not a perfect square}$$

Alternately, “Some multiples of 3 are not perfect squares.”

General forms for universal and existential quantification:

$$\forall x \in X, P(x)$$

$$\exists x \in X \mid P(x)$$

$\forall x \in \emptyset, P(x)$ is **always (vacuously) true**.

$\exists x \in \emptyset \mid P(x)$ is **always false**

$$\sim (\forall x \in X, P(x))$$

$$\equiv \sim (P(x_1) \wedge P(x_2) \wedge \dots)$$

$$\equiv \sim P(x_1) \vee \sim P(x_2) \vee \dots \quad \text{By DeMorgan's Law}$$

$$\equiv \exists x \in X \mid \sim P(x)$$

T	S	R	Q	P
K	L	M	N	O
J	I	H	G	F
E	D	C	B	A

1. Bob passed through *P*.
2. Bob passed through *N*.
3. Bob passed through *M*.
4. If Bob passed through *O*, then Bob passed through *F*.
5. If Bob passed through *K*, then Bob passed through *L*.
6. If Bob passed through *L*, then Bob passed through *K*.

Let X be the routes through the maze, that is,
 $X = \{CBGFONQR, CDILMNQR, CDIJKLMNQR\}$

Let $P(x) =$ route x contains L ,
 $Q(x) =$ route x contains K .

Consider $\forall x \in X, P(x) \rightarrow Q(x)$.

X	$P(x)$	$Q(x)$	$P(x) \rightarrow Q(x)$
$CBGFONQR$			
$CDILMNQR$			
$CDIJKLMNQR$			

T	S	R	Q	P
K	L	M	N	O
J	I	H	G	F
E	D	C	B	A

For next time:

Do Exercises 3.6.(2, 5-9) and 3.7.(20-24)

Read Section 4.1

Take quiz