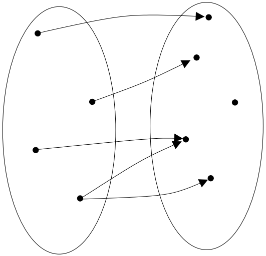
Chapter 6 outline:

- Introduction, function equality, and anonymous functions (Wednesday)
- Image and inverse images (**Today**)
- Function properties and composition (next week Monday)
- Map, reduce, filter (next week Wednesday)
- Cardinality (next week Friday)
- Countability (week-after Monday, Nov 18)
- Review (week-after Wednesday, Nov 20)
- Test 3, on Ch 5 & 6 (week-after Friday, Nov 22)

Today:

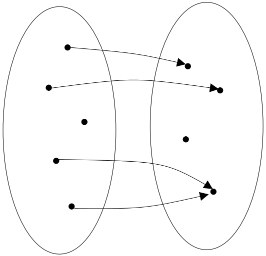
- Review definitions from last time
- New definitions: image and inverse image
- Programming
- Proofs

A relation f from X to Y is a function (written $f : X \rightarrow Y$) if $\forall\ x \in X$,
(1) $\exists\ y \in Y \mid (x, y) \in f$, and (2) $\forall\ y_1, y_2 \in Y$, $(x, y_1), (x, y_2) \in f \rightarrow y_1 = y_2$.



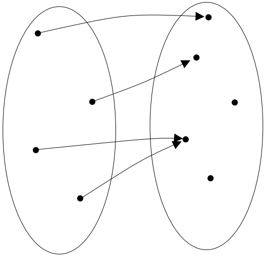
Not a function.

(There's a domain element that is related to two things.)



Not a function.

(There's a domain element that is not related to anything.)

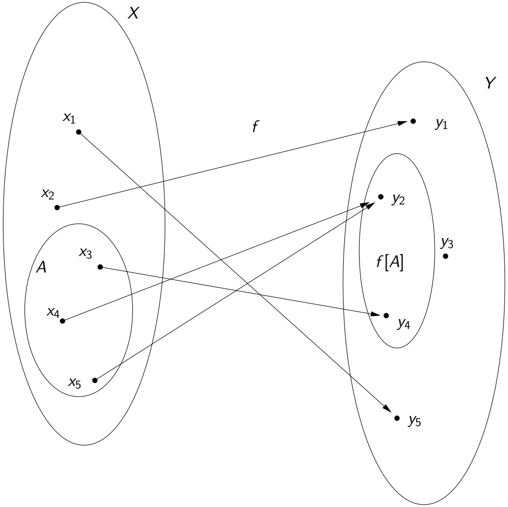


A function.

(It's OK that two domain elements are related to the same thing and one codomain element has nothing related to it.)

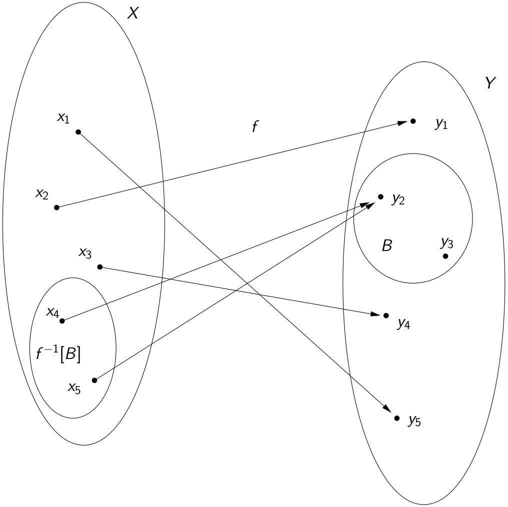
Image

$$f[A] = \{y \in Y \mid \exists\, x \in A \text{ such that } f(x) = y\}$$



Inverse image

$$f^{-1}[B] = \{x \in X \mid f(x) \in B\}$$



Lemma 6.2. If $f : X \to Y$, then $f[\emptyset] = \emptyset$.

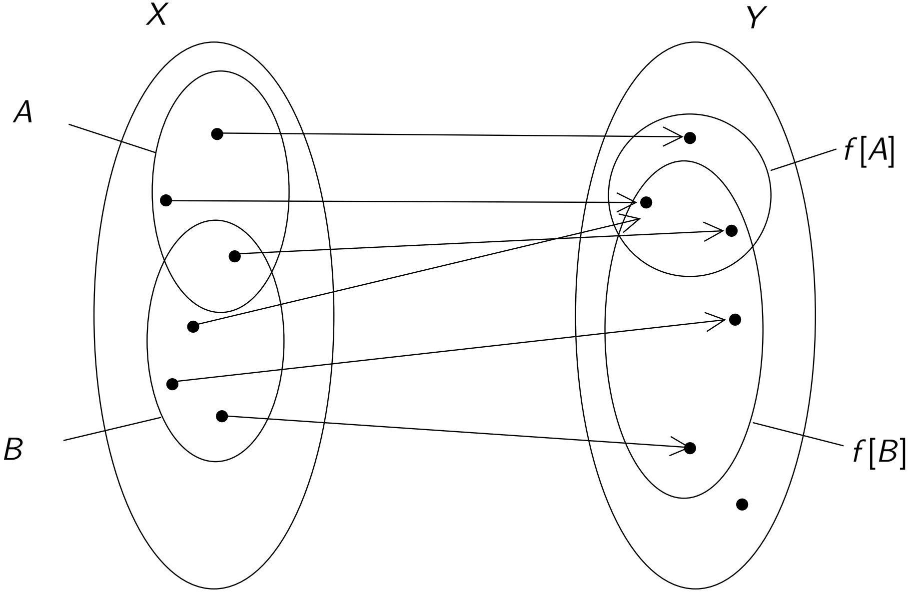
Lemma 6.3. If $f : X \to Y$, $A \subseteq X$, and $A \neq \emptyset$, then $f[A] \neq \emptyset$.

Lemma 6.4. If $f : X \to Y$, then $f^{-1}[\emptyset] = \emptyset$.

We might expect the following, but *it's not true*:

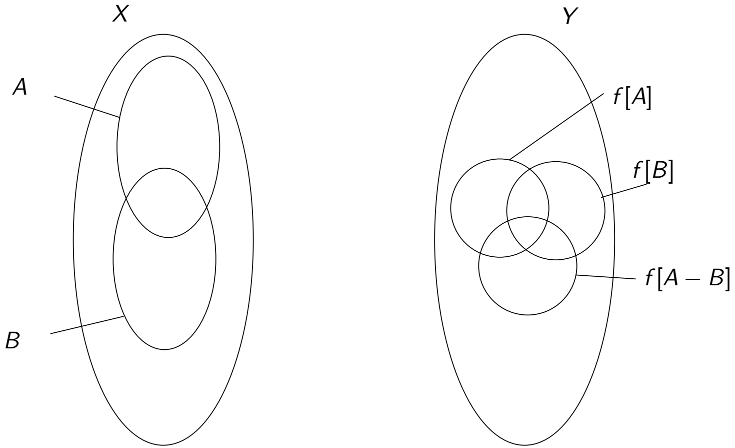
Lemma XXXX. If $f : X \to Y$, $A \subseteq Y$, and $A \neq \emptyset$, then $f^{-1}[A] \neq \emptyset$.

Ex 6.2.1. If $A, B \subseteq X$, then $f[A \cap B] \subseteq f[A] \cap f[B]$.



Ex 6.2.3. If $A, B \subseteq X$, then $f[A - B] \subseteq f[A] - f[B]$?

Consider this picture of X and Y:



Ex 6.2.3. If $A, B \subseteq X$, then $f[A - B] \subseteq f[A] - f[B]$?

Attempted proof. Suppose $A, B \subseteq X$ and $y \in f[A - B]$. By definition of image, there exists $x \in A - B$ such that $f(x) = y$.

Ex 6.2.3. If $A, B \subseteq X$, then $f[A - B] \subseteq f[A] - f[B]$?

Attempted proof. Suppose $A, B \subseteq X$ and $y \in f[A - B]$. By definition of image, there exists $x \in A - B$ such that $f(x) = y$.

By definition of difference, $x \in A$, and $x \notin B$. By definition of image, $f(x) \in f[A]$.

Ex 6.2.3. If $A, B \subseteq X$, then $f[A - B] \subseteq f[A] - f[B]$?

Attempted proof. Suppose $A, B \subseteq X$ and $y \in f[A - B]$. By definition of image, there exists $x \in A - B$ such that $f(x) = y$.

By definition of difference, $x \in A$, and $x \notin B$. By definition of image, $f(x) \in f[A]$.

So, also by definition of image, $f(x) \notin f[B]$. Right?

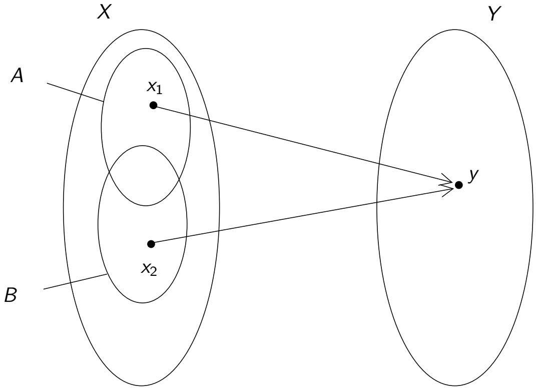
Ex 6.2.3. If $A, B \subseteq X$, then $f[A - B] \subseteq f[A] - f[B]$?

Attempted proof. Suppose $A, B \subseteq X$ and $y \in f[A - B]$. By definition of image, there exists $x \in A - B$ such that $f(x) = y$.

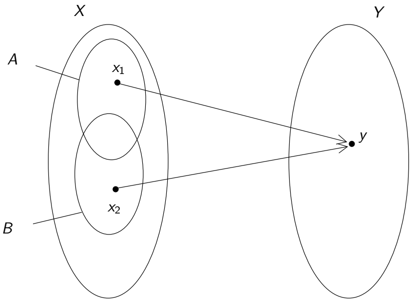
By definition of difference, $x \in A$, and $x \notin B$. By definition of image, $f(x) \in f[A]$.

So, also by definition of image, $f(x) \notin f[B]$. Right?

NO!



Ex 6.2.3. If $A, B \subseteq X$, then $f[A - B] \subseteq f[A] - f[B]$?



Let $X = \{x_1, x_2\}$, $Y = \{y\}$, $A = \{x_1\}$, and $B = \{x_2\}$.

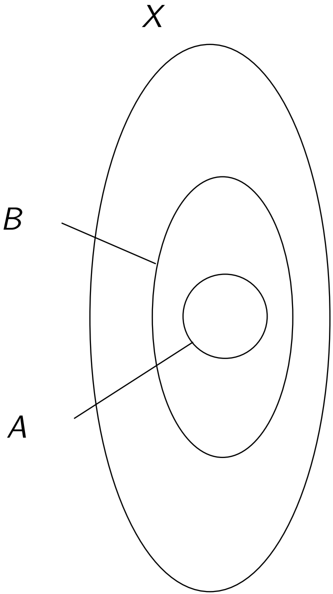
Let $f = \{(x_1, y), (x_2, y)\}$.

Then $f[A-B] = f[\{x_1\}-\{x_2\}] = f[\{x_1\}] = \{y\}$.

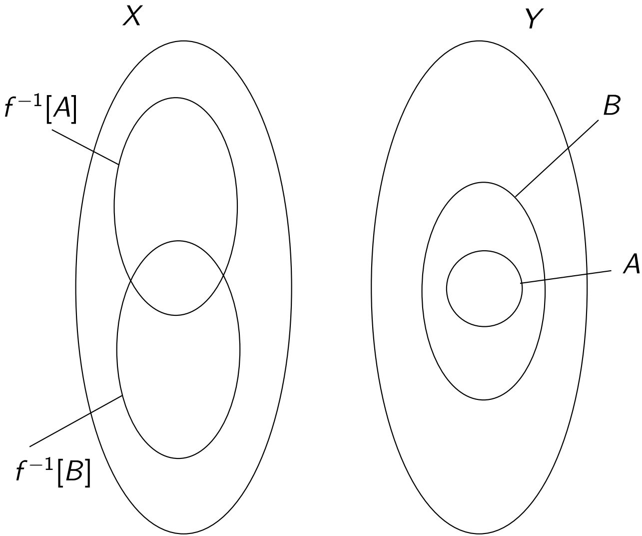
Moreover, $f[A] - f[B] = \{y\} - \{y\} = \emptyset$.

So $f[A - B] \not\subseteq f[A] - f[B]$

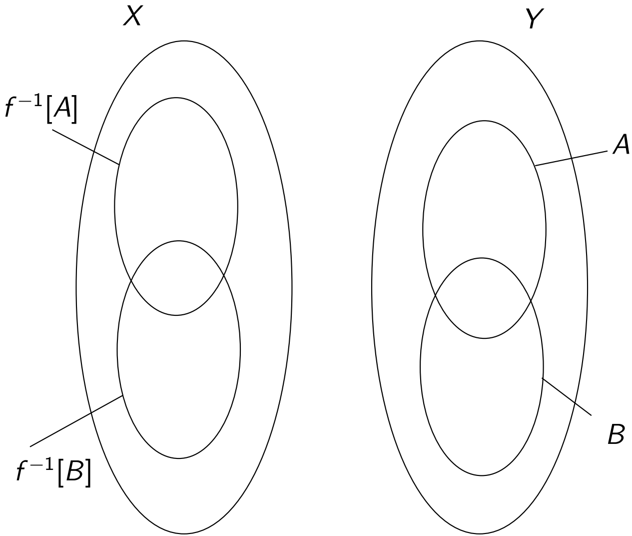
Ex 6.2.4. If $A \subseteq B \subseteq X$, then $f[B] = f[B - A] \cup f[A]$.



Ex 6.2.6. If $A \subseteq B \subseteq Y$, then $f^{-1}[A] \subseteq f^{-1}[B]$.



Ex 6.2.7. If $A, B \subseteq Y$, then $f^{-1}[A \cup B] = f^{-1}[A] \cup f^{-1}[B]$.



For next time:

Do Exercises 6.2.(2, 5, 8, 9, 10).

No programming problems this time; there is an all-programming assignment coming up.

See Canvas for hint on 6.2.5

Read Section 6.(3 & 4)