Chapter 7, Hash tables:

- General introduction; separate chaining (week-before Wednesday)
- Open addressing (week-before Friday)
- Hash functions (last week Monday)
- Perfect hashing (Monday)
- Hash table performance (**Today**)
- (Start Ch 8, Strings, Thursday (in lab) and Friday)

Today:

- Elements of hashtable performance
- Separate chaining performance
- Open addressing performance

End-of-semester important dates

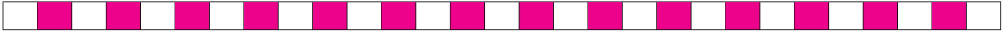
- Thurs, Dec 5: Last “normal” lab
- Mon, Dec 9: Last project assigned
- Tues, Dec 10: Last “normal” running of project grading script
- Wed, Dec 11: Test 3 & 4 Review sheet distributed, Test 4 practice problems made available.
- Thurs, Dec 12: Review lab (pick practice problems for Test 4)
- Fri, Dec 13, AM: “Two-minute warning” running of project grading script (Canvas gradebook will not be updated—see project report in your turn-in file)
  Note that Fri, Dec 13 is the *Last Day of Classes*.
- Fri, Dec 13, midnight: Official project deadline
- Sat, Dec 14, when I wake up: Permissions to turn-in folders turned off
- Mon, Dec 16: Project grading script run for final/semester grades
- Wed, Dec 18, 10:30am-12:30pm: Tests 3 and 4 (in lab)

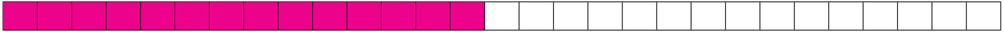
	Find	Insert	Delete
Unsorted array	$\Theta(n)$	$\Theta(1)$ [$\Theta(n)$]	$\Theta(n)$
Sorted array	$\Theta(\lg n)$	$\Theta(n)$	$\Theta(n)$
Linked list	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$
Balanced BST	$\Theta(\lg n)$	$\Theta(1)$ [$\Theta(\lg n)$]	$\Theta(1)$ [$\Theta(\lg n)$]
What we want	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

Find	Search the data structure for a given key
Insert	Add a new key to the data structure
Delete	Get rid of a key and fix up the data structure

`containsKey()`	Find
`get()`	Find
`put()`	Find + insert
`remove()`	Find + delete

$$
\left.
\begin{array}{rll}
 & O(1) & c_0 \\
 & O(1) & c_0 \\
 & O(1) & c_0 \\
 & \vdots & \\
 & O(1) & c_0 \\
\text{rehash} \longrightarrow & O(n) & c_1 + c_2 n \\
 & O(1) & c_0 \\
 & \vdots & \\
 & O(1) & c_0
\end{array}
\right\}
\begin{array}{rcl}
T(n) & = & (n-1)c_0 + c_1 + c_2 n \\
 & = & (c_0 + c_2)n + c_1 - c_0 \\
 & = & \Theta(n)
\end{array}
$$





$$
\begin{aligned}
&\frac{(n+1)+n+(n-1)+\cdots+3+2+\overbrace{1+\cdots+1}^{m-n}}{m} \\
&\quad = \frac{m+n+(n-1)+\cdots+2+1}{m} && \text{the initial } m \text{ accounting for the last probe in each case} \\
&\quad = \frac{m}{m}+\frac{(n+1)\cdot\frac{n}{2}}{m} && \text{as an arithmetic series} \\
&\quad \approx 1+\frac{(n+1)\cdot\frac{n}{2}}{2\cdot n} && \text{since } m \text{ is about } 2\cdot n \\
&\quad = 1+\frac{n+1}{4} && \text{by cancellation}
\end{aligned}
$$

$$\frac{[(s_0+1)+s_0+(s_0-1)+\cdots+2]+\cdots+1+\cdots 1}{m} = 1+\frac{\sum_{i=0}^{\gamma-1}\sum_{j=1}^{s_i} j}{m}$$

What is the probability that a miss k requires at least i probes?

Conditional probability

$P(X \mid Y)$: What is the probability of event X in light of event Y?

$$
\begin{aligned}
P(X \wedge Y) &= P(X) \cdot P(X \mid Y) \\
P(X_0 \wedge X_1 \wedge \cdots \wedge X_{N-1}) &= P(X_0) \cdot P(X_1 \mid X_0) \cdot P(X_1 \mid X_0 \wedge X_1) \cdots P(X_{N-1} \mid X_0 \wedge \cdots \wedge X_{N-2})
\end{aligned}
$$

$$P(\,T[h(k)+1] \neq \texttt{null} \mid T[h(k)] \neq \texttt{null}) = \frac{n-1}{m-1}$$

The probability that a miss requires at least i probes:

$$\begin{aligned} & \frac{n}{m} \cdot \frac{n-1}{m-1} \cdots \frac{n-i+2}{m-i+2} \\ \leq \quad & \left(\frac{n}{m}\right)^{i-1} && \text{since } n < m \\ \leq \quad & \alpha^{i-1} && \text{by substitution} \end{aligned}$$

$$
\begin{aligned}
\sum_{i=1}^{m} i \cdot P\binom{\text{it takes}}{i \text{ probes}} &= \sum_{i=1}^{m} i \cdot \left(P\begin{pmatrix}\text{it takes}\\ \text{at least } i\\ \text{probes}\end{pmatrix} - P\begin{pmatrix}\text{it takes at}\\ \text{least } i+1\\ \text{probes}\end{pmatrix} \right) && \\
&= \sum_{i=1}^{m} P\begin{pmatrix}\text{it takes}\\ \text{at least } i\\ \text{probes}\end{pmatrix} && \text{by telescoping} \\
&\leq \sum_{i=1}^{m} \alpha^{i-1} && \text{by the previous result} \\
&\leq \sum_{i=1}^{\infty} \alpha^{i-1} && \text{since } m < \infty \\
&= \sum_{i=0}^{\infty} \alpha^{i} && \text{by a change of variable} \\
&= \frac{1}{1-\alpha} && \text{by geometric series}
\end{aligned}
$$

Is the following assumption true for linear probing?

$$P(T[h(k)+1] \neq \texttt{null} \mid T[h(k)] \neq \texttt{null}) = \frac{n-1}{m-1}$$

In general, is the following assumption true for a probing strategy?

$$P(T[\sigma(k,1)] \neq \texttt{null} \mid T[\sigma(k,0)] \neq \texttt{null}) = \frac{n-1}{m-1}$$

What is the difference between

Each array index is equally likely to be the hash of a given key.	vs	Each array position is equally likely to be occupied.

Linear probing is biased towards clustering:

x	Number of buckets with exactly x previous buckets filled	Number of filled buckets with exactly x previous buckets filled	Probability that a bucket is filled if exactly x previous buckets are filled.
0	97	48	.495
1	48	22	.458
2	22	12	.545
3	12	7	.583
4	7	4	.571
5	4	3	.75
6	3	2	.667
7	2	2	1
8	2	0	0

Expected number of probes for a miss in a hashtable using linear probing (from Knuth):

$$\frac{1}{2} \cdot \left(1 + \frac{1}{(1-\alpha)^2}\right)$$

After n calls to `put()` with unique keys, no removals, consider **average chain length** over all keys (low is good), **percent of keys that are in their ideal location** (high is good), and **length of the longest chain** (low is good)

	n	Linear probing			Quadratic probing			Double hashing		
Surnames	1000	2.092	64.7%	31	1.421	75.8%	9	2.327	65.2%	31
Mountains	1360	1.568	73.8%	17	1.729	65.8%	11	1.770	73.4%	16
Mountains (height)	1360	1.932	75.1%	99	1.882	68.9%	18	1.830	72.4%	13
Chemicals	663	1.517	75.0%	16	1.729	65.5%	10	1.701	75.5%	9
Chemicals (symbol)	663	1.885	71.0%	20	1.837	66.4%	13	1.798	72.7%	12
Books	718	1.419	76.7%	8	1.659	70.0%	11	1.656	75.8%	8
Books (ISBN)	718	1.542	74.4%	21	1.670	67.8%	15	1.724	74.5%	10
Random strings	5000	1.544	77.6%	49	1.735	69.9%	37	1.598	78.1%	13
Random strings	5000	1.531	77.1%	35	1.729	69.8%	28	1.593	77.9%	12
Random strings	5000	1.643	77.5%	76	1.754	68.6%	29	1.590	78.1%	13

Coming up:

Do **Open addressing hashtable** *project (due this past Mon, Dec 2)*
Do **Perfect hashing** *project (due mon, Dec 9)*

Due **Wed, Dec 4** *(end of day)*
Re-read the last part of Section 7.3
Take quiz

Due **Fri, Dec 6** *(end of day)*
Read Section 8.1
Do Exercises 8.(4 & 5)
Take quiz

Due **Mon, Dec 9** *(end of day)*
Read Section 8.2
(No quiz or practice problems)