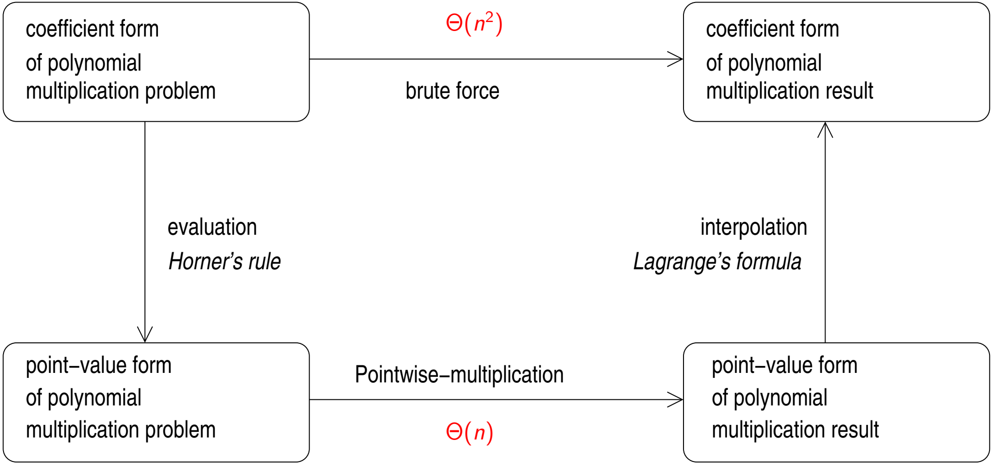
II. Topics / A. Fast Fourier transform

- Intoduction to premise and problem (last week Wednesday)
- The complex roots of unity (last week Friday)
- FFT algorithm details (**today**)

Today:

- Review of FFT in general
- Finish theorems
- Construct algorithm
- Inspect code


coefficient form
of polynomial
multiplication problem
$\Theta(n^2)$
brute force
coefficient form
of polynomial
multiplication result
evaluation
Horner's rule
interpolation
Lagrange's formula
point–value form
of polynomial
multiplication problem
Pointwise–multiplication
$\Theta(n)$
point–value form
of polynomial
multiplication result

In general, the *principal nth root of unity* is $\omega_n = e^{\frac{2\pi i}{n}}$

The n complex nth roots of unity are $\omega_n^0, \omega_n^1, \ldots, \omega_n^{n-1}$.

Note that $\omega_n = \omega_n^1$ and $\omega_n^0 = \omega_n^n = 1$.

Note also that $\omega_n^k = e^{\frac{2\pi i}{n}k} = e^{\frac{2k\pi i}{n}} = \cos(\frac{2k\pi}{n}) + i\sin(\frac{2k\pi}{n})$

Cancellation lemma. (30.3)
For any integers $n \geq 0$, $k \geq 0$, and $d > 0$, $\omega_{dn}^{dk} = \omega_n^k$.

Proof. $\omega_{dn}^{dk} = (\omega_{dn})^{dk} = (e^{\frac{2\pi i}{dn}})^{dk} = (e^{\frac{2\pi i}{n}})^k = \omega_n^k$. $\Box$

Corollary to above. (30.4)
For any even integer $n > 0$, $\omega_n^{\frac{n}{2}} = \omega_2 = -1$

Proof. Let m be such that $n = 2m$. Then $\omega_n^{\frac{n}{2}} = \omega_{2m}^m = \omega_2 = -1$. $\Box$

Cancellation lemma rewritten.
If d is a common divisor of n and k, then $\omega_n^k = \omega_{\frac{n}{d}}^{\frac{k}{d}}$.

Halving lemma.
If $n > 0$ is even, then the squares of the n complex nth roots of unity are the $\frac{n}{2}$ complex $\frac{n}{2}$th roots of unity.

To compute $A(x)$ on $\omega_n^1, \omega_n^2, \ldots, \omega_n^n$,

$$A_{even}(\omega_{\frac{n}{2}}^1),\ A_{even}(\omega_{\frac{n}{2}}^2),\ \ldots A_{even}(\omega_{\frac{n}{2}}^{\frac{n}{2}})$$

and $$A_{odd}(\omega_{\frac{n}{2}}^1),\ A_{odd}(\omega_{\frac{n}{2}}^2),\ \ldots A_{odd}(\omega_{\frac{n}{2}}^{\frac{n}{2}})$$

and so $$A(\omega_n^k) = A_{even}(\omega_{\frac{n}{2}}^{\frac{k}{2}}) + \omega_n^k A_{odd}(\omega_{\frac{n}{2}}^{\frac{k}{2}})$$

Coming up:

Stop *working on the DP + Greedy problemn set, turn in what you have if you haven't already.*

Do Problem 30-1 *as the FFT problem set. See elaboration in separate document on Canvas. Due Mon, Oct 28.*

Skim *LP Chapter 1,* **read** *(carefully) LP Section 2.(1 & 2).*
(See next slide)

From Canvas:

From this point on, all readings and exercises come from Lewis and Papadimitriou ("LP"), unless otherwise noted. Note that LP exercises are named chapter.section.exercise, as in 2.1.3, exercise 3 in section 1 of chapter 2. Some exercises have parts, and some of those parts have subparts, such as 2.1.5.a.ii.

Skim ch 1. We won't talk about it in class, and it should be all review from DMFP and other math courses. Just take note of this book's notational conventions etc. In particular, some parts Sections 1.(7 & 8) will likely be wholly new for a lot of you. You don't need to have all of it completely internalized by class, but you should make sure you get the definition of language on pg 44, and that you will know where to refer to when the other terms in those sections come up in later chapters.

Then read 2.(1 & 2). We will probably talk only about 2.1, but reading 2.2 now will cut down on the reading for next time.

(No exercises. For the next few daily-work assignments, the reading and the exercises will be "misaligned", that is, the exercise will go with the reading for the previous class day.)